Fluctuations in R&D Investment and Long-run Growth: The Role of the Size Distribution of Innovating Firms

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Abstract

R&D investment drives productivity growth. Therefore, its fluctuations over the business cycle affect long-run dynamics. I show that taking into account the size distribution of innovating firms generates new insights on this link. I write an endogenous growth model with heterogeneous firms assuming, in line with empirical evidence, that small firms have a relatively higher innovation capacity than large firms. In addition, the model predicts that small firms reduce R&D more than large firms after negative aggregate shocks, especially financial ones. These differences between firms generate two novel predictions at the aggregate level. First, a financial shock leads to a rightward shift in the size distribution of innovating firms. This amplifies the shock's effect on productivity growth and makes it persistent over time. Second, there is a positive correlation between the share of an industry's innovations done by small firms, the industry's average productivity growth rate and the volatility of that growth rate. Firm-level empirical evidence supports the greater reaction of small firms' R&D to negative aggregate shocks, and the model's aggregate predictions are consistent with a line of stylized facts.

Keywords: R&D, Innovation, Heterogeneous Firms, Size Distribution, Endogenous Persistence

JEL Codes: E32, O31, O33

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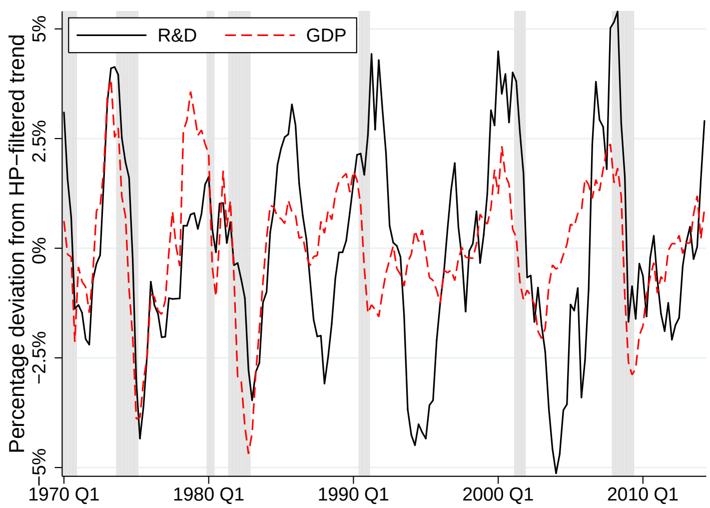
1 Introduction

Private firms' investments in Research and Development (R&D) create a link between short-run and long-run economic developments. Indeed, while R&D is a key determinant of the long-run productivity level, it also reacts systematically to business cycle fluctuations.

Figure 1 illustrates this second point by plotting the deviations of quarterly real private R&D investment and of quarterly real GDP in the United States from their Hodrick-Prescott filtered trends. It shows that fluctuations in both variables are positively correlated over the last decades. In particular, R&D generally falls below trend during NBER recessions. These decreases in R&D slow down innovation, and may therefore adversely affect the level of productivity and output even long after the actual recession has ended.

Figure 1: R&D and GDP fluctuations in the United States, 1970-2014



Source: US Bureau of Economic Analysis, National Income and Product Accounts. Shaded areas indicate NBER Recessions. Time series are computed as percentage deviations from a trend calculated using a Hodrick-Prescott filter with smoothing parameter $\lambda = 1600$.

The procyclicality of R&D, widely documented in empirical studies[1], has stimulated a large amount of research because it challenged some important theoretical priors. For instance, Joseph Schumpeter, who introduced innovation into economic analysis, had argued that recessions prepared the ground for new innovation waves.[2] Other researchers had stressed that the opportunity cost of allocating resources away from current production falls in recessions, giving firms an incentive to do more R&D (Aghion and Saint-Paul (1993)). Therefore, the recent literature has mainly focused on identifying mechanisms which can overturn these countercyclical forces and explain the procyclicality observed in the data. The leading explanations to date

[1] See for example Comin and Gertler (2006), Barlevy (2007), and Ouyang (2011), among many others.

[2] Schumpeter thought that short-run fluctuations in innovation and long-run growth were inseparable parts of the "capitalist development process". In *The Theory of Economic Development* (Schumpeter (1934)), he argued that economic crises create the conditions for new innovation waves by lowering factor prices and creating a stock of idle resources. Matsuyama (1999) and Francois and Lloyd-Ellis (2003) formalised this idea in general equilibrium models.

include financial constraints that bind more in recessions (Stiglitz (1993), Aghion et al. (2010), Garcia-Macià (2014)) and the procyclicality of private profits from R&D (Comin and Gertler (2006), Barlevy (2007)).
This theoretical literature can explain why R&D falls during recessions, and provides a framework for analysing the long-run effects of this fall. However, it has, to the best of my knowledge, not yet taken into account firm-level heterogeneity. In particular, it has not considered differences between small and large firms. This is potentially an important omission, as empirical studies suggest that there are substantial differences between the type and the amount of R&D done by small and large firms. For instance, Cohen and Klepper (1996) argue that small innovating firms generate more innovations per dollar of R&D spent. Akcigit and Kerr (2010) show that small innovating firms spend relatively more on R&D than large ones, and that their innovations are more likely to be major technological advances. These findings suggest that the size distribution of innovating firms matters, and that taking it into account may generate new insights on how R&D dynamics link the short and the long run.

I explore this hypothesis by writing a partial equilibrium endogenous growth model with heterogeneous firms and aggregate shocks. The model describes an industry in which a continuum of firms produces a fixed mass of differentiated goods. A firm's size is measured as the number of goods it produces. Firms can improve productivity through two types of R&D. Radical R&D creates radical innovations, enabling the innovating firm to increase the frontier productivity for a good which it does not produce yet (and therefore allowing it to displace that good's current producer). Incremental R&D creates incremental innovations, enabling the innovating firm to increase the frontier productivity of goods which it already produces.
Innovation in my model has two features that are crucial for my results. First, radical innovation triggers creative destruction which shapes the size distribution. Firms grow (or enter the industry) doing radical innovation, and they shrink (and eventually exit) when displaced by the radical innovations of others. Second, I assume that the relative innovation capacity of small firms is higher than the one of large firms. That is, when there are no shocks and firms realise all their innovation opportunities, overall productivity growth in the industry is higher if the fixed mass of goods is produced by a large mass of small firms rather than by a small mass of large firms.[3]
The model's basic structure builds on Akcigit and Kerr (2010), who themselves build on Klette and Kortum (2004). However, while these papers also analyse an endogenous growth model with heterogeneous firms, they focus on a balanced growth path, while I analyse the dynamics triggered by aggregate shocks. In particular, I consider shocks to the level of spending on all goods of the industry (aggregate demand shocks)

[3]This feature of my model is motivated by the aforementioned empirical evidence, suggesting that small firms spend relatively more and more productively on R&D and are relatively more likely to generate major technological advances. This strongly suggests that their relative innovation capacity (and thus their relative contribution to aggregate productivity growth) is higher than the one of large firms.

and to firms' ability to borrow (financial shocks). My model delivers three main predictions.

Prediction 1. On average, small firms reduce R&D more than large ones after a negative aggregate shock.

This holds for financial shocks because small firms have lower cash holdings, and for aggregate demand shocks because they have lower profit margins. The combination of small firms' greater sensitivity to negative aggregate shocks and their higher relative innovation capacity generates two novel aggregate predictions, which show that taking into account firm-level heterogeneity in R&D indeed yields new insights on aggregate dynamics.

Prediction 2. A financial shock leads to a rightward shift in the size distribution of innovating firms. This amplifies the shock's effect on productivity growth and makes it persistent over time.

A financial shock does not affect large firms, but forces many small ones to abandon radical R&D, as their cash holdings are insufficient to pay the R&D cost. Thus, entry falls, creative destruction slows down and the size distribution shifts persistently to the right. As large firms have a relatively lower innovation capacity than small ones, productivity growth remains depressed even once the actual shock has vanished.

Prediction 3. There is a positive correlation between the share of an industry's innovations done by small firms, the industry's average productivity growth rate and the volatility of that growth rate.

More precisely, any development that shifts the size distribution of innovating firms to the left (without having a direct effect on productivity) ends up increasing the average rate of productivity growth, as there are now more innovative small firms, but also its volatility, as these small firms are more sensitive to negative aggregate shocks. This suggests a positive correlation between the share of innovations done by small firms (or any other measure of the relative importance of small innovating firms), average growth and volatility.

The firm-level and aggregate predictions of my model are consistent with empirical evidence. Using a German firm-level dataset, I show that during the 2007-2009 Great Recession (which may be interpreted as a joint aggregate demand and financial shock), the median small firm reduced R&D more than the median large firm, even though its sales fell less. As a result, a significant gap in the relative R&D intensity of small and large firms opened up in the crisis years, in line with Prediction 1.

At the aggregate level, Prediction 2 is consistent with the experience of several countries during the Great Recession. For instance, both in Germany and in the United States, there was a (small) rightward shift in the size distribution of innovating firms. Furthermore, empirical studies indicate a persistent fall in productivity growth after the recession in many OECD countries.[4] Finally, I show that as implied by Prediction 3, the

[4]In the United States, there is a debate about whether this slowdown is due to the Great Recession. Fernald (2014) and Hall (2014) dispute this and claim that productivity growth fell even before the recession started. In other OECD countries, however, the direct impact of the Great Recession on productivity growth appears more significant (Ball (2014)).

average and the standard deviation of productivity growth are positively correlated across industries in the United States manufacturing sector. The lack of extensive industry-level data on the size distribution of innovating firms makes it impossible to formally test whether this new stylized fact is linked to differences in the size distribution. The available data is however consistent with this hypothesis.

The remainder of the paper is structured as follows. Section 2 describes my model's assumptions and Section 3 derives and discusses its predictions. Section 4 presents the empirical evidence and Section 5 concludes.

2 R&D investment, firm size and aggregate shocks

In the early 1990s, endogenous growth models formalized the idea that R&D drives productivity growth. In an important contribution, Klette and Kortum (2004) extend these models to allow for a rich set of firm dynamics. In their model, innovations are homogenous and innovation capacity is exactly proportional to firm size. Thus, the size distribution of innovating firms does not affect aggregate productivity growth. Akcigit and Kerr (2010) and Acemoglu and Cao (2010) depart from this model by assuming that small firms' and/or entrants' innovations are more productive than those of large incumbents. In this case, the size distribution matters, and size-dependent subsidies or taxes may affect aggregate productivity growth.[5]

My model builds on these contributions, but introduces aggregate shocks instead of just considering a balanced growth path. This enables me to study the long-run impact of the R&D fluctuations triggered by these shocks. Sections 2.1 to 2.4 lay out the model's assumptions.

2.1 The basic structure

I model an industry in partial equilibrium. Time is discrete ($t \in \mathbb{N}$). In every period t, the industry produces a fixed set of differentiated goods, indexed on the interval $[0, 1]$. The next two subsections describe the demand and the supply side of the model in greater detail.

2.1.1 Demand

I assume that aggregate spending on all goods of the industry follows an exogenous Markov process $(S_t)_{t\in\mathbb{N}}$. Aggregate spending can take two values, S_H and S_L (with $S_H > S_L$). A fall of spending from S_H to S_L is a negative aggregate demand shock. In every period t, a continuum of identical consumers allocates aggregate spending across the goods of the industry. Consumers aim to maximise their utility, given by

$$C_t = \exp\left(\int_0^1 \ln c_t(j)\, dj\right), \tag{1}$$

[5] Acemoglu et al. (2013) analyse this question in greater detail, focusing on the resource allocation among innovating firms.

where $c_t(j)$ denotes the quantity of good j consumed in period t. They take the prices of differentiated goods and aggregate spending as given.

2.1.2 Firms

The set of differentiated goods is produced by a continuum of atomistic firms, each individual firm producing a finite number of goods. I denote by $m_{n,t}$ the mass of firms producing n different goods in period t. The number of goods produced is my measure of firm size. As all firms are potential innovators, the firm size distribution coincides with the size distribution of innovating firms.[6] All goods of the interval $[0,1]$ are produced, so that in every period t, $\sum_{n=1}^{+\infty} n m_{n,t} = 1$.

Apart from incumbent firms, there also exists a mass $m_{0,t}$ of potential entrants which do not produce in period t. I assume that the relative mass of potential entrants with respect to incumbents is constant:

$$\frac{m_{0,t}}{m_t} = \psi, \tag{2}$$

where $m_t \equiv \sum_{n=1}^{+\infty} m_{n,t}$ is the mass of incumbent firms and $\psi > 0$ is a fixed parameter which may be interpreted as an inverse measure of barriers to entry into R&D.

Firms produce goods with a constant returns to scale technology using labour. For any firm i,

$$\forall j \in [0,1], \;\; q_t^i(j) = a_t^i(j)\, l_t^i(j), \tag{3}$$

where $q_t^i(j)$ stands for the output of good j produced by firm i in period t, $l_t^i(j)$ for labour used in production and $a_t^i(j)$ for firm i's productivity for good j. Note that a firm's productivity may differ across goods. Labour supply to firms is given by the reduced-form function $L(w_t)$, increasing in the wage w_t. The wage (just as aggregate spending and prices) is expressed in money units, due to the model's partial equilibrium structure.

Firms maximize the expected net present value (NPV) of profits earned over their existence. They store all the cash they earn (with a constant rate of return set to 0 for simplicity), and only pay dividends upon exit. Potential entrants start out without any cash.

Finally, I assume that there is Bertrand competition on the market for each differentiated good. This implies that only the firm with the highest productivity for a given good produces it in equilibrium, and that this firm earns positive profits only if its productivity is strictly higher than that of all other firms. Firms therefore have an incentive to improve their productivity by investing in R&D. The next section describes their R&D technology in greater detail.

[6] In the theoretical sections of this paper, I therefore use both terms interchangeably. In real-world datasets, however, the two concepts do not coincide, as many firms never innovate.

2.2 R&D investment and innovation

Firms may improve productivity by creating radical or incremental innovations.[7]

Radical innovations. In every period t, every firm (whether incumbent or potential entrant) receives a radical innovation opportunity with a fixed probability α. This opportunity allows the firm to create a radical innovation and increase the frontier productivity for some good j which it currently does not produce, if it pays the radical R&D cost f_R. More precisely, if $a_t(j)$ denotes the frontier productivity for good j in period t,[8] a radical innovation enables the firm to produce good j with productivity $\gamma a_t(j)$ $(\gamma > 1)$ from period $t+1$ onwards. Radical innovation triggers creative destruction, and thereby changes the firm size distribution: by becoming the most productive producer for good j, the innovating firm displaces the incumbent producer of that good. A firm which has lost all its goods in this way exits.[9] The good to which a radical innovation applies is drawn from a pool of "contestable" goods, containing one (randomly selected) good of every incumbent firm. Thus, a firm can neither gain nor lose more than one good per period.

Incremental innovations. Incumbent firms do not only receive radical innovation opportunities, but can also improve the frontier productivity of goods they already produce by incremental innovation. Paying an incremental R&D cost f_I in period t enables them to increase their productivity for all their non-contestable goods by a factor δ $(\delta > 1)$ in period $t+1$.

I assume that the productivity increases implied by radical and incremental innovation hold

$$\alpha \ln \gamma > \ln \delta. \tag{4}$$

This inequality states that if all goods were subject to radical innovation (would see their productivity increase by a factor γ with probability α), aggregate productivity would increase more than if all goods were subject to incremental innovation (would see their productivity increase by a factor δ for sure).[10] In sum, this implies that radical innovation contributes relatively more to productivity growth than incremental innovation, justifying the names of both innovation types.[11]

[7] This follows the terminology of Acemoglu and Cao (2010).

[8] That is, $a_t(j) = \max_i a_t^i(j)$.

[9] Exit is permanent, meaning that an exiting firm is not in the set of potential entrants next period.

[10] To see this, note that as utility is a Cobb-Douglas aggregator of the differentiated goods (see Equation (1)), a natural expression for the industry's aggregate productivity (abstracting from labour misallocation, which will be discussed in Section 3.1.3 below) is $A_t = \exp\left(\int_0^1 \ln(a_t(j))\right)$. If all goods were subject to radical innovation, a fraction α of them would see their productivity increased by a factor γ, and aggregate productivity would be multiplied by γ^α. If all goods were subject to incremental innovation, aggregate productivity would be multiplied by δ. $\gamma^\alpha > \delta$ holds if and only if $\alpha \ln \gamma > \ln \delta$.

[11] To illustrate the nature of radical and incremental innovation, some examples from the car industry may be useful. A recent radical innovation in this industry is the Tesla Roadster (the first long-range all-electrical sports car, developed by a Silicon Valley start-up). The Tesla Roadster is radical both because of the large technological advance it represents and because of its potential to displace conventional sports car producers. In contrast, General Motors' introduction of the OnStar system (a teleassistance system which, for example, automatically contacts an emergency hotline in case of an accident) may be considered an incremental innovation: it is not a major technological advance, and it applies to all of General Motors' cars (improving already produced goods without displacing other firms).

For simplicity, I also assume that one period after an innovation is introduced for a given good, imitation allows all firms to produce with a productivity that is arbitrarily close to the one of the innovator. This limits the profits from innovation to one period.[12]

R&D costs for both types of innovation must be paid in period t, while innovations appear only in period $t+1$. This creates a need for finance for all firms which cannot pay R&D costs with their cash holdings. The next section explains the conditions under which firms can borrow to finance R&D.

2.3 Finance and financial shocks

Financing conditions exogenously fluctuate between two possible states of the world. In the "normal" state, firms can borrow as much as needed for R&D projects with a non-negative NPV. In the "crisis" state, firms cannot borrow at all. Thus, if financing conditions are in the crisis state in period t, every firm must hold

$$c_t \geq \mathbf{1}_{R,t} f_R + \mathbf{1}_{I,t} f_I, \tag{5}$$

where c_t stands for the firm's cash holdings in period t (after production and before deciding on R&D investment) and $\mathbf{1}_{R,t}$ and $\mathbf{1}_{I,t}$ are indicator variables for the firm's decision to do radical R&D (conditional on getting a radical innovation opportunity) or incremental R&D in period t. A switch from normal to crisis financing conditions is a financial shock.

To prevent occasionally binding financial constraints from inducing precautionary savings, I assume that firms can forecast aggregate demand one period ahead. As profits from innovation are limited to one period, this ensures that a project with positive NPV decided upon in period t never lowers the firm's cash holdings in period $t+1$.

2.4 The innovation capacity of small and large firms

Before proceeding to solve the model, it is useful to shortly discuss the key features of the innovation process generated by my assumptions. Two features in particular are crucial for my results.

First, radical innovation drives firm dynamics and shapes the firm size distribution. Firms grow (or enter the industry) doing radical innovation, and they shrink (and eventually exit) when displaced by the radical innovations of others. This is a common feature in heterogeneous firm models in the tradition of Klette and Kortum (2004), in line with the extensive empirical evidence on firm-level creative destruction.

[12] As the innovator retains an infinitesimal productivity advantage, it remains the only producer of the good as long as it is not displaced by a radical innovation. However, to simplify notation, I assume in the following that imitators can produce with the exact frontier productivity.

Second, in my model, the relative innovation capacity of small firms is higher than that of large firms. This feature is the joint effect of three assumptions.

(a) Radical innovation capacity is independent of firm size.

(b) Incremental innovation capacity is a linear function of firm size.

(c) Radical innovations contribute relatively more to productivity growth than incremental ones.

These assumptions imply that when firms realise all their innovation opportunities, the relative contribution of a firm of size n to aggregate productivity growth is $\frac{\alpha \ln \gamma + (n-1) \ln \delta}{n}$.[13] Under Equation (4), this relative contribution is clearly decreasing in firm size n. Therefore, when firms realise all their innovation opportunities, a leftward shift in the firm size distribution increases productivity growth.[14]

This key feature of my model is supported by empirical evidence. The early empirical literature, launched by Schumpeter's (1934, 1942) conjectures on the role of small and large firms for innovation, shows that small firms generate more innovations per dollar of R&D than large ones (Cohen and Klepper (1996)). In the same vein, Kortum and Lerner (2000) argue that young and small firms promoted by venture capital funds account for a disproportionate share of their industry's innovations. Finally, recent empirical studies assess the relative technological advances represented by small and large firms' innovations, using extensive patent datasets. Akcigit and Kerr (2010) show that the average patent filed by a small firm receives more citations and that major innovations are relatively more frequent in small firms. Ewens and Fons-Rosen (2013) show that upon leaving an established IT firm and creating a small start-up, patents filed by founders are on average of better quality and more likely to represent pioneering innovations than patents filed by their former co-workers. Overall, these results suggest that small firms indeed have a greater innovation capacity than large ones, as implied by my model's assumptions.[15]

In the next section, I solve for the model's equilibrium and derive its firm-level and aggregate predictions.

3 Equilibrium and predictions

3.1 Equilibrium and dynamic laws of motion

I first determine the equilibrium on goods and labour markets in period t. Then, I solve for firms' optimal R&D choices and derive the laws of motion of aggregate variables and of the firm size distribution.

[13] This relative contribution is defined as the aggregate productivity growth of the industry in case all firms were of size n.

[14] On these matters, my model is similar to the one of Akcigit and Kerr (2010). They also assume that radical (in their terminology, "exploration") innovation capacity is independent of firm size and show that this implies a relatively greater innovation capacity of small firms.

[15] There is no direct evidence on the relative contribution of small and large firms to industry productivity growth, due to obvious measurement problems. However, Acs and Audretsch (1990) do find that an industry's innovative activity decreases in its concentration level, in line with my assumptions.

3.1.1 Demand, prices and market clearing

Consumers maximize their utility from the consumption of differentiated goods defined in (1), taking aggregate spending S_t and prices $(p_t(j))_{j\in[0,1]}$ as given. This yields the standard demand system

$$\forall j \in [0,1], \quad c_t(j) = \frac{S_t}{p_t(j)}. \tag{6}$$

Goods market clearing implies $c_t(j) = q_t(j)$ for every j. Thus, defining aggregate output as $Q_t \equiv \exp\left(\int_0^1 \ln q_t(j)\, dj\right)$ gives

$$Q_t = C_t = \frac{S_t}{P_t}, \quad \text{with } P_t = \exp\left(\int_0^1 \ln p_t(j)\, dj\right). \tag{7}$$

Bertrand competition implies that the producer of any good sets a price equal to the marginal cost of the second most productive firm.[16] Moreover, imitation implies that the second most productive firm in period t has a productivity equal to the one of the most productive firm in period $t-1$. Therefore,

$$\forall j \in [0,1], \quad p_t(j) = \frac{w_t}{a_{t-1}(j)}. \tag{8}$$

Substituting (8) into (6) gives $q_t(j) = \frac{S_t a_{t-1}(j)}{w_t}$. Therefore, the labour demand of the firm producing good j is $l_t(j) = \frac{S_t a_{t-1}(j)}{w_t a_t(j)}$, and the labour market clearing condition is

$$L(w_t) = \frac{S_t}{w_t} \int_0^1 \frac{a_{t-1}(j)}{a_t(j)} dj. \tag{9}$$

Equations (6) to (9) define the equilibrium values of output, prices and wages in period t. Note that they are independent of current R&D decisions (because R&D does not use labour). However, R&D affects future productivities, and thereby future values of output, prices and wages.

3.1.2 R&D investment

To determine firms' R&D investment decisions, I first compute their profits from producing a radically or incrementally improved good. The imitation assumption implies that profits are limited to the period in which the innovation is introduced.

Consider first a good j whose productivity is improved by radical innovation in period $t+1$. The radical innovation enables the innovating firm to charge a markup γ over marginal cost (it can produce with unit cost

[16] More precisely, the equilibrium price is the minimum between the marginal cost of the second most productive firm (the limit price) and the monopoly price. As the demand function defined in (6) has a price elasticity of 1, the monopoly price tends towards positive infinity and the limit price is always the equilibrium price.

$\frac{w_{t+1}}{\gamma a_t(j)}$ and all other firms, with unit cost $\frac{w_{t+1}}{a_t(j)}$). Therefore, its profit from production equals $\left(1-\frac{1}{\gamma}\right)S_{t+1}$. Likewise, the profit from producing an incrementally improved good in period $t+1$ is $\left(1-\frac{1}{\delta}\right)S_{t+1}$. Profits increase in the markup (which is equal to the increase in the frontier productivity) and do not depend on the identity of the improved good.

I can now solve for the R&D policy function of firms in period t. The policy function depends on two endogenous state variables: the number of goods produced by the firm, n_t, and its cash holdings after production, c_t. Even though R&D decisions are binary, they generate a complicated dynamic programming problem. Therefore, I impose from now on three additional parameter restrictions on innovations' NPVs that deliver an explicit solution.

Restriction 1. $\left(1-\frac{1}{\gamma}\right)S_L - f_R > 0$. The NPV of radical innovation is always positive.

Restriction 2. $\left(1-\frac{1}{\delta}\right)S_H - f_I > 0$. When aggregate demand is high, the NPV of incremental innovation is positive for a firm with one non-contestable good.

Restriction 3. $\left(1-\frac{1}{\gamma}\right)S - f_R > (n_U^*-1)\left(1-\frac{1}{\delta}\right)S - f_I$ for $S \in \{S_L, S_H\}$.
$n_U^* = \left\lceil \frac{f_R+f_I}{(1-\frac{1}{\gamma})S_L - f_R} \right\rceil$, where $\lceil . \rceil$ is the ceiling function, assigning to each real number x the smallest natural number larger than x. This restriction implies that the NPV of radical innovation is always greater than the NPV of incremental innovation for firms small enough to be constrained in crisis financing conditions.

The role of these parameter restrictions becomes clear when analysing the R&D policy functions they deliver. I first analyse the policy function of a firm in a period with normal financing conditions.[17]

R&D decisions for normal financing conditions. Consider a firm with n_t goods and cash holdings c_t. Then, if financing conditions are normal in period t,

$$\mathbf{1}_{R,t} = 1,$$

$$\mathbf{1}_{I,t} = 1 \Leftrightarrow n_t \geq n_{I,t}^* = \left\lceil 1 + \frac{f_I}{\left(1-\frac{1}{\delta}\right)S_{t+1}} \right\rceil.$$

Under normal financing conditions, all firms spend on radical R&D if they get a radical innovation opportunity. This follows immediately from Restriction 1, ensuring that the NPV of a radical innovation is always positive.[18] Firms spend on incremental R&D if and only if its NPV is non-negative. For a firm with n_t goods,

[17] I only give an intuitive account of firms' policy functions in the main text. Proofs are provided in Appendix A.1.

[18] This restriction considerably simplifies the firms' problem, as it excludes situations in which the negative cash flow from a radical innovation must be compared to the positive value it generates by increasing firm size (thus allowing the firm to survive longer and to generate more innovations in the future). The restriction excludes from the outset that radical R&D reacts to demand shocks. This is extreme, but captures the fact that high profit (radical) innovations are less likely to be abandoned than low profit (incremental) ones if future demand is low.

the NPV of incremental innovation on its $n_t - 1$ non-contestable goods is $(n_t - 1)\left(1 - \frac{1}{\delta}\right) S_{t+1} - f_I$. This value increases in n_t (showing that large firms benefit from economies of scope for incremental innovation) and is non-negative if and only if n_t is larger than the threshold size $n^*_{I,t}$ defined above. $n^*_{I,t}$ depends on aggregate demand next period and may therefore take two possible values, which I denote by $n^*_{I,H}$ and $n^*_{I,L}$. Restriction 2 implies that $n^*_{I,H} = 2$: when aggregate demand is high and financing conditions are normal, firms realise all their innovation opportunities.[19]

With crisis financing conditions in period t, the desired R&D choices of firms do not change, but they may not be feasible any more for firms with low cash holdings.

n^*_U, defined in Restriction 3, is the threshold size for firms to be forever unconstrained: firms with n^*_U goods or more have accumulated enough cash to be unaffected by crisis financing conditions.[20] The threshold size n^*_U is derived by remarking that the lowest possible cash level for a firm with n goods is $n\left(\left(1 - \frac{1}{\gamma}\right) S_L - f_R\right)$, as the firm must have earned at least n times the NPV of radical innovation (which is at worst $\left(1 - \frac{1}{\gamma}\right) S_L - f_R$). All firms with less than n^*_U goods continue to spend on radical and incremental innovation if and only if their cash holdings are sufficient. Restriction 3 implies that if these firms want to do radical and incremental R&D, but have only enough resources to do one of the two, they choose radical R&D.[21] Thus, decisions with crisis financing conditions can be summarized as follows.

R&D decisions for crisis financing conditions. Consider a firm with n_t goods and cash holdings c_t. Then, if there are crisis financing conditions in period t,

$$\mathbf{1}_{R,t} = 1 \Leftrightarrow c_t \geq f_R$$

$$\mathbf{1}_{I,t} = 1 \Leftrightarrow n_t \geq n^*_{I,t} \text{ and } \begin{cases} c_t \geq f_R + f_I \text{ or } f_I \leq c_t < f_R & \text{if the firm gets a radical innovation opportunity} \\ c_t \geq f_I & \text{else} \end{cases}$$

Firms' R&D policy functions directly illustrate the model's first important prediction, regarding small and large firms' reactions to a negative aggregate shock.

Prediction 1. On average, small firms reduce R&D more than large ones after a negative aggregate shock.

As firms do not pay dividends before exit, average cash holdings increase in firm size. A financial shock, which forces firms to finance R&D with their cash holdings, therefore especially affects small firms. Aggregate

[19] This feature is inessential for my results. However, if some small firms with non-contestable goods would never do incremental R&D, the condition on the relative productivity improvements triggered by radical and incremental R&D given in Equation (4) must be strengthened somewhat to ensure that in the absence of negative shocks, small firms still contribute relatively more to productivity growth than large ones.

[20] As the NPV of all realised innovations is positive, and fully collected one period after the R&D investment is made, cash holdings never fall. Therefore, a firm which is unconstrained once is in fact unconstrained forever.

[21] This restriction is not important for my results, but considerably simplifies calculations.

demand shocks also hit small firms (among those with non-contestable goods) harder. As they benefit relatively less from economies of scope, their NPV of incremental innovation is low and turns negative if demand falls, while the one of the largest firms always remains positive.

Together with the assumptions ensuring that the relative innovation capacity of small firms is higher than the one of large firms, Prediction 1 is key for my model's aggregate predictions. Before getting to these, however, I need to complete the model's solution.

3.1.3 The dynamic laws of motion

The two previous sections show how equilibrium quantities and prices depend on current productivity, and how firms decide on R&D, which determines future productivity. It is now time to put the pieces together and to determine the model's dynamic laws of motion.

Denote by $u_{n,t}^{f}$ the fraction of firms of size n which can finance in period t an R&D cost f. By definition, $u_{n,t}^{f}=1$ for every n and f if financing conditions are normal in period t. Under crisis financing conditions, $u_{n,t}^{f}$ is equal to the fraction of firms of size n which have (after production in period t) cash holdings larger or equal to f. Then, the mass of radical innovations due to R&D in period t is

$$R_t=\alpha\sum_{n=0}^{+\infty}u_{n,t}^{f_R}m_{n,t}. \tag{10}$$

Likewise, the mass of incremental innovations due to R&D in period t is

$$I_t=\alpha\sum_{n=n_{I,t}^{*}}^{+\infty}(n-1)\left(u_{n,t}^{f_R+f_I}+\max\left(0,u_{n,t}^{f_I}-u_{n,t}^{f_R}\right)\right)m_{n,t}+(1-\alpha)\sum_{n=n_{I,t}^{*}}^{+\infty}(n-1)\,u_{n,t}^{f_I}m_{n,t}. \tag{11}$$

The productivity of radically improved goods increases by a factor γ from period t to $t+1$, the one of incrementally improved goods increases by a factor δ, and the one of all other goods remains unchanged. Therefore, using Equations (7) and (8), I get a law of motion for the aggregate price level P_t:[22]

$$\frac{P_{t+1}}{P_t}=\frac{w_{t+1}}{w_t}\exp\left(-\left(R_{t-1}\ln\gamma+I_{t-1}\ln\delta\right)\right). \tag{12}$$

Using Equation (8), I can deduce a law of motion for aggregate output:

$$\frac{Q_{t+1}}{Q_t}=\frac{S_{t+1}}{S_t}\frac{w_t}{w_{t+1}}\exp\left(R_{t-1}\ln\gamma+I_{t-1}\ln\delta\right). \tag{13}$$

[22] Equation (12) is formally derived in Appendix A.2.

Finally, defining aggregate productivity as $A_t \equiv \frac{Q_t}{L_t}$, it comes that

$$\frac{A_{t+1}}{A_t} = \exp\left(R_t \ln\gamma + I_t \ln\delta\right)\left(\exp\left(\left(R_{t-1}-R_t\right)\ln\gamma + \left(I_{t-1}-I_t\right)\ln\delta\right)\frac{1-R_{t-1}\left(1-\frac{1}{\gamma}\right)-I_{t-1}\left(1-\frac{1}{\delta}\right)}{1-R_t\left(1-\frac{1}{\gamma}\right)-I_t\left(1-\frac{1}{\delta}\right)}\right). \tag{14}$$

Aggregate productivity growth obviously depends on increases in productivity generated by radical and incremental R&D, as captured by the first factor in Equation (14). However, there is also a more indirect source of variation. R&D investment creates markup dispersion (unimproved goods are sold at marginal cost, improved ones at a markup γ or δ) which misallocates labour and depresses aggregate productivity.[23] The second factor in Equation (14) indicates that if markup dispersion increases between periods t and $t+1$ (through R&D changes between periods $t-1$ and t), aggregate productivity falls.

Equations (10) and (11) show that the firm size distribution determines innovation masses. The firm size distribution itself may change between periods as a result of radical innovation. Indeed, an incumbent producing n goods in period t may be in three possible states in period $t+1$.

(a) It may produce $n+1$ goods, if it does radical R&D and does not lose its contestable good. By the law of large numbers, the fraction of firms with n goods doing radical R&D is $\alpha u_{n,t}^{f_R}$. I denote by d_t the probability that a firm loses its contestable good (in the following, I refer to this as the destruction rate). Receiving a radical R&D investment opportunity and losing a contestable good are independent events, so that finally, a fraction $\alpha u_{n,t}^{f_R}\left(1-d_t\right)$ of firms producing n goods transitions to producing $n+1$ goods.

(b) It may keep producing n goods. This happens if it does not do radical R&D and does not lose its contestable good (which is the case for a fraction $\left(1-\alpha u_{n,t}^{f_R}\right)\left(1-d_t\right)$ of firms), or if it does radical R&D, but also loses its contestable good (which is the case for a fraction $\alpha u_{n,t}^{f_R} d_t$ of firms).

(c) It may produce $n-1$ goods.[24] This happens if it does not do radical R&D and loses its contestable good, which is the case for a fraction $\left(1-\alpha u_{n,t}^{f_R}\right) d_t$ of firms.

Potential entrants cannot lose goods by definition, and a fraction $\alpha u_{0,t}^{f_R}$ of them enters each period.

Equation (15) summarizes the changes in the size distribution between periods t and $t+1$.

$$\begin{aligned} & m_{1,t+1} = \alpha u_{0,t}^{f_R} m_{0,t} + \left(\left(1-\alpha u_{1,t}^{f_R}\right)\left(1-d_t\right) + \alpha u_{1,t}^{f_R} d_t\right) m_{1,t} + \left(1-\alpha u_{2,t}^{f_R}\right) d_t m_{2,t} \\ \forall n \geq 2, \quad & m_{n,t+1} = \alpha u_{n-1,t}^{f_R}\left(1-d_t\right) m_{n-1,t} + \left(\left(1-\alpha u_{n,t}^{f_R}\right)\left(1-d_t\right) + \alpha u_{n,t}^{f_R} d_t\right) m_{n,t} \\ & \qquad + \left(1-\alpha u_{n+1,t}^{f_R}\right) d_t m_{n+1,t} \end{aligned} . \tag{15}$$

The destruction rate d_t is endogenous. It equals the ratio between the mass of radical innovations and the

[23] Epifani and Gancia (2011) and Peters (2011) analyse the static and dynamic implications of this markup dispersion.
[24] For a firm with $n = 1$ good in period t, this means exit.

mass of incumbent firms:[25]

$$d_t = \frac{R_t}{m_t}. \tag{16}$$

This completes the description of equilibrium. In the next section, I briefly analyse the model's balanced growth path.

3.2 The balanced growth path

I define the balanced growth path as the model's solution when financing conditions are normal and aggregate demand is high in every period t. It provides a useful starting point before turning to the analysis of the fluctuations triggered by aggregate shocks.

On the balanced growth path, all firms innovate up to their full innovation capacity. Thus, a fraction α of each size group of firms does a radical innovation every period, the destruction rate is constant, and there is a unique invariant firm size distribution, given by[26]

$$\forall n \geq 1, \;\; m_n = \left(\frac{\psi}{(1-\alpha)(1+\psi)}\right)^2 \left(\frac{1-\alpha(1+\psi)}{(1-\alpha)(1+\psi)}\right)^{n-1}, \tag{17}$$

$$m_0 = \frac{\psi^2}{(1-\alpha)(1+\psi)}. \tag{18}$$

Assuming the initial firm size distribution in period $t = 0$ is the invariant one, all aggregate variables grow at a constant rate. In particular, using Equations (10) and (11), it is easy to show that innovation masses are constant over time and hold $R = \frac{\alpha\psi}{1-\alpha}$ and $I = \frac{1-\alpha(1+\psi)}{(1-\alpha)(1+\psi)}$. This implies

$$\forall t \in \mathbb{N}, \;\; \frac{Q_{t+1}}{Q_t} = \frac{A_{t+1}}{A_t} = \exp\left(R \ln \gamma + I \ln \delta\right) \qquad \text{and} \qquad \frac{P_{t+1}}{P_t} = \exp\left(-\left(R \ln \gamma + I \ln \delta\right)\right). \tag{19}$$

Wages, labour supply and the efficiency of the labour allocation remain constant over time.

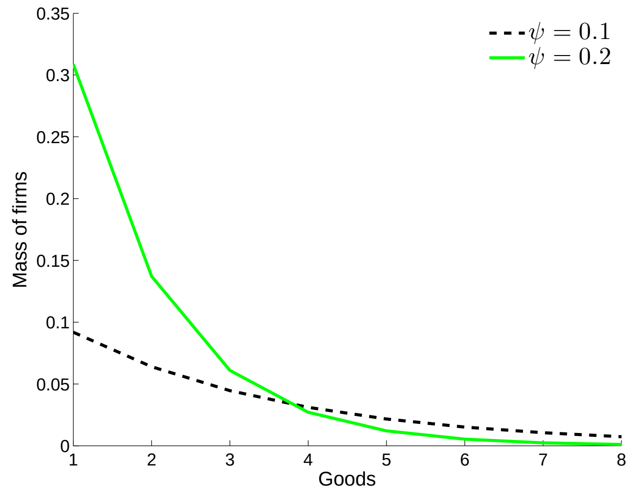
The shape of the invariant firm size distribution is partly determined by the relative mass of potential entrants ψ. An increase in ψ shifts the distribution to the left.[27] This is intuitive: an increase in the mass of potential entrants increases actual entry and accelerates creative destruction. Incumbents are less likely to grow large, and a greater share of goods is produced by entrants or small firms. Figure 2 illustrates this by plotting the invariant firm size distribution for $\alpha = 0.7$ and two different values of ψ.

[25] The model's parameter values need to be restricted such that in every period, $d_t \leq 1$. It can easily be shown that $\alpha(1+\psi) < 1$ is a sufficient condition for this, and I assume from now on that it holds.

[26] The invariant size distribution is formally derived in Appendix A.3.

[27] See Appendix A.3 for a proof of this claim.

Figure 2: The invariant firm size distribution for different values of ψ



As small firms have a higher relative innovation capacity than large firms, such shifts in the size distribution are not neutral, but increase aggregate productivity growth. I now analyse the implications of this non-neutrality when aggregate demand and financing conditions are subject to shocks.

3.3 Impulse responses to financial and aggregate demand shocks

In this section, I analyse the impact of a financial and/or aggregate demand shock for an industry on the balanced growth path. That is, I assume that the initial firm size distribution is the invariant one, and financing conditions are normal and aggregate demand is high from period 0 to period $T-1$ and from period $T+1$ to $+\infty$. Only in the crisis period T, there is a financial and/or aggregate demand shock.

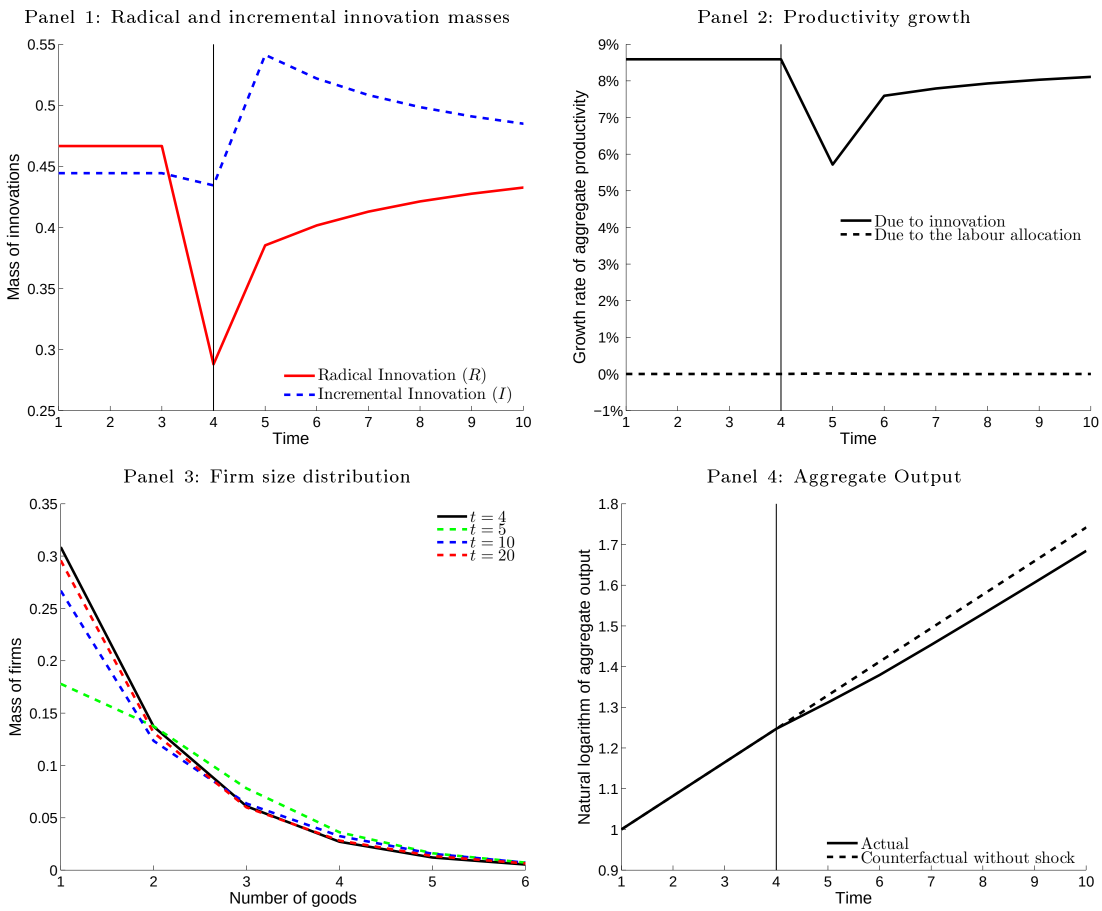
3.3.1 Impulse responses to a financial shock

I assume first that in period T, there are crisis financing conditions, but aggregate demand is at its high level S_H. Figure 3 shows the impulse responses of the most important variables to this financial shock. The vertical line in Panels 1, 2 and 4 indicates the period in which the shock hits.

Panel 1 shows that when the financial shock occurs, the masses of radical and incremental innovations fall, as potential entrants and a fraction of small incumbents do not have sufficient cash holdings to finance R&D.[28] Lower innovation leads to a drop in productivity growth (see Panel 2) and a permanent fall in the long-run level of output. This mechanism has been repeatedly highlighted by representative firm models. However, taking into account firm-level heterogeneity yields a further amplification and persistence effect which is absent from the earlier literature.

[28] I determine the fractions $u_{n,t}^{f}$ of unconstrained firms numerically. Details are provided in Appendix A.4.

Figure 3: Impulse responses to a financial shock



Notes: The crisis shock hits at $T = 4$. Parameter values used for computing impulse responses are given in Table 7 in Appendix A.5.

As the financial shock disrupts radical innovation, it keeps potential entrants from entering and a fraction of small firms from expanding. Therefore, it shifts the firm size distribution to the right, as illustrated in Panel 3. As small firms' innovation capacity is relatively higher than the one of large firms, this implies that productivity growth remains below its balanced growth path level, as shown in Panel 2. The shift in the firm size distribution (and therefore the fall in productivity growth[29]) is persistent over time: even as entry and creative destruction resume in the aftermath of the shock, it takes time to replace the "lost generation" of small firms which could not enter or expand during the crisis period. The fact that productivity growth remains depressed even when the financial shock has vanished greatly amplifies the shock's effect on the long-run level of output. This is illustrated in Panel 4, which plots the actual path of output against a counterfactual one that would have prevailed in the absence of any shock. The panel shows that the permanent output loss from the shock increases for several periods after the shock has passed, as long as productivity growth remains

[29] Productivity growth is almost entirely explained by micro-level productivity improvements triggered by R&D. As shown in Panel 2, the effect of changes in the labour allocation is negligible.

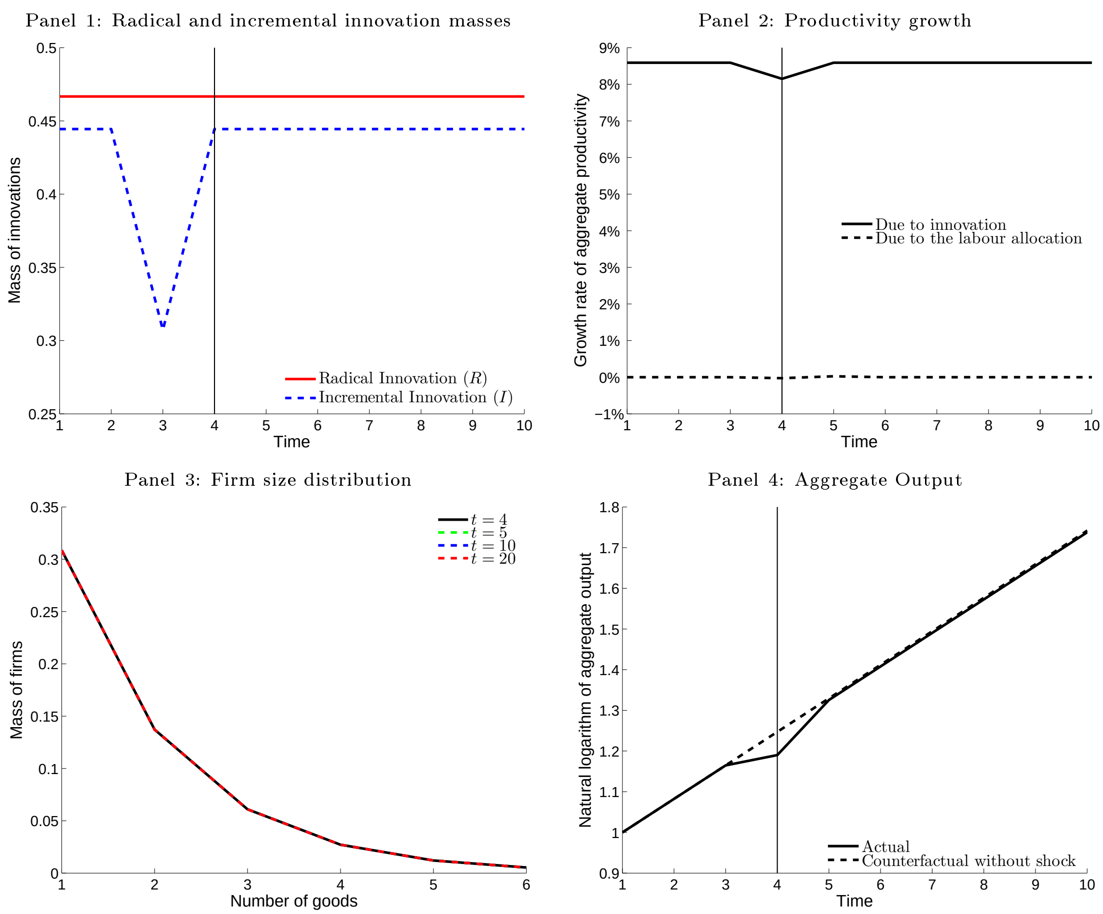
below its balanced growth path level.

Finally, note that as long as the size distribution remains shifted to the right, radical innovation is depressed (as the overall mass of firms is below its balanced growth path level) and incremental innovation is stimulated (as the rightward shift in the size distribution increases the fraction of non-contestable goods).

3.3.2 Impulse responses to an aggregate demand shock

I now assume that in period T, aggregate demand falls to its low level S_L, but financing conditions are normal. The impulse responses to this aggregate demand shock are shown in Figure 4.

Figure 4: Impulse responses to an aggregate demand shock



Notes: See Figure 3.

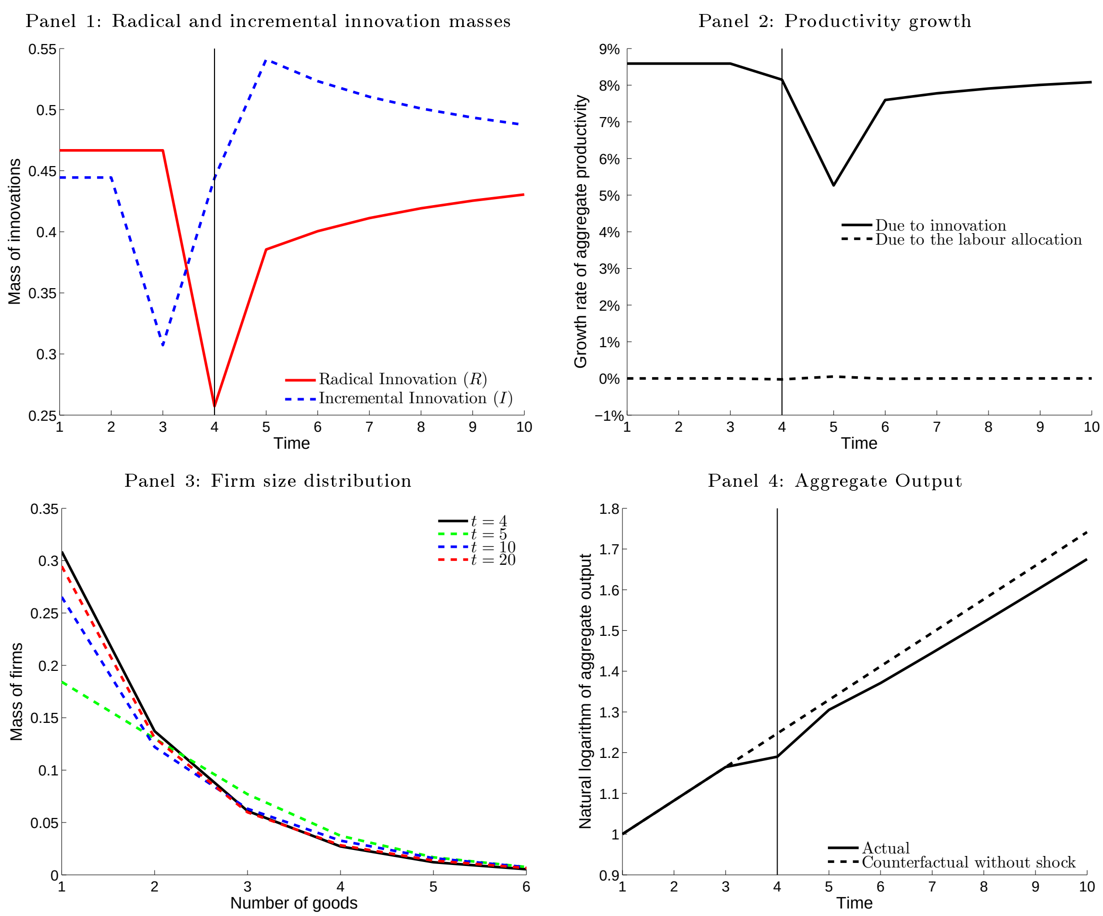
Panel 1 shows that radical innovations are unaffected by the shock, as they are profitable even with low aggregate demand. Incremental innovations, however, are not profitable any more for small firms with low profit margins. Therefore, the mass of incremental innovations falls (one period before the shock hits, as firms forecast aggregate demand). This fall is not persistent: when aggregate demand is back to its balanced growth path level, incremental innovation and overall productivity growth also jump back.

An aggregate demand shock has no persistent effect on innovation masses and productivity growth because it does not affect the firm size distribution (see Panel 3). Thus, its only long-run effect on output and productivity is a small permanent loss due to the fall of R&D and innovation on impact. Moreover, Panel 4 shows that unlike a financial shock, an aggregate demand shock also has a short-run effect on output. The fall in demand reduces wages, and this reduces labour supply and output. This effect is reversed in the period after the shock, when spending, wages and labour supply increase again.

3.3.3 Impulse responses to a joint financial and aggregate demand shock

Figure 5 shows impulse responses when both a financial and an aggregate demand shock hit in period T.

Figure 5: Impulse responses to a joint aggregate demand and financial shock



Notes: See Figure 3.

Figure 5 illustrates the model's prediction for the impact of the 2007-2009 Great Recession, which may be seen as a joint financial and aggregate demand shock. The effects of a joint shock simply combine those of the two shocks taken separately. On impact, incremental innovation falls in anticipation of the fall in aggregate demand, and radical innovation falls because financial constraints are binding for entrants and a fraction of small firms. The fall in radical innovation induces a rightward shift in the firm size distribution which depresses productivity growth even after the shock has vanished. Therefore, even after recovering from the short-run effect of the aggregate demand shock, output growth remains depressed as well (see Panel 4).

It is worth noting that although financial constraints play an important role in my model, its workings are very distinct from the financial accelerator mechanism, as it has been described in Bernanke and Gertler (1989) or, in a model with heterogeneous firms and creative destruction, Caballero and Hammour (2005). In these models, financial constraints are permanent. A transitory aggregate demand (or productivity) shock persistently lowers firms' profits. This makes financial constraints more likely to bind in the following periods and persistently depresses investment. In the impulse responses shown in Figure 5, the fall in profits due to the aggregate demand shock has only a minor effect: it increases the mass of constrained firms as long as financing conditions are in the crisis state. Once financing conditions have normalised, however, the fall in profits becomes irrelevant, as firms do not need them to finance R&D. In sum, my model emphasizes a persistence mechanism which is both distinct from and complementary to the financial accelerator.

The main conclusion of this section is summarized in Prediction 2.

Prediction 2. A financial shock leads to a rightward shift in the size distribution of innovating firms. This amplifies the shock's effect on productivity growth and makes it persistent over time.[30]

In the next section, I generalise the analysis of the firm size distribution's role further by considering an arbitrary sequence of aggregate demand and financial shocks.

3.4 Growth, volatility and the firm size distribution

The firm size distribution in my model is endogenous, but it is also affected by exogenous parameters such as α and ψ, which may be interpreted as fixed industry characteristics. The analysis from the previous section suggests that if there is a change in an exogenous factor shifting the invariant size distribution to the left (for instance, an increase in ψ, the relative mass of potential entrants) the industry's balanced growth rate increases, but the fall in this growth rate after a negative aggregate shock increases as well.[31] These considerations lead to Prediction 3.

[30] Note that this mechanism is not limited a priori to financial shocks. It equally applies to any other transitory shock which disrupts radical innovation in small firms and therefore shifts the size distribution of innovating firms to the right.

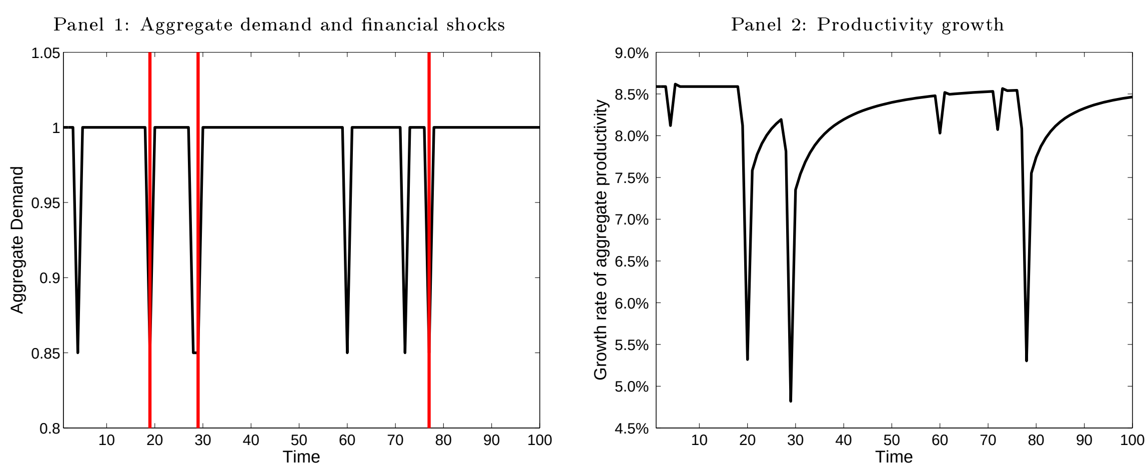
[31] When the firm size distribution is more left-skewed, there is a greater mass of firms which are constrained after a financial shock. For an aggregate demand shock, the analysis is less clear-cut. Indeed, firms that abandon incremental innovation after an aggregate demand shock are small, but they are not the smallest firms (firms with only one good, for instance, never do incremental innovation). However, if a leftward shift in the firm size distribution does not only increase the mass of firms with one good, but also the mass of slightly larger firms, it also makes the industry more sensitive to an aggregate demand shock.

Prediction 3. There is generally a positive correlation between the share of an industry's innovations done by small firms, the industry's average productivity growth rate and the volatility of that growth rate.

Note that Prediction 3 does not hold unconditionally. In particular, it assumes that all industry-level characteristics which jointly affect the size distribution and productivity growth have been controlled for.[32] Furthermore, it presumes that industries are hit by the same sequence of shocks, and that these shocks are not too frequent. Indeed, if an industry with a very left-skewed firm size distribution were constantly hit by financial shocks, its average productivity growth could be lowered decisively. However, as financial crises are rare events in reality, assuming infrequent shocks appears reasonable.

I illustrate Prediction 3 by simulating the model for a random sequence of shocks. For this, I assume that transition probabilities for the aggregate demand Markov chain are $p_{HL} = 0.1$ and $p_{LH} = 0.8$. Furthermore, I assume financial shocks are correlated with aggregate demand: crisis financing conditions occur with probability $p_F = 0.25$ for any period with low aggregate demand and do not occur if aggregate demand is high.[33] I then draw a sequence of shocks for 100 periods and calculate the model's solution for it (assuming the industry starts on the balanced growth path). Panel 1 of Figure 6 provides an example for a sequence of shocks. The solid line plots aggregate demand over time, while vertical bars mark periods with financial shocks. Panel 2 shows the corresponding path of productivity growth. It is fully in line with the intuitions developed in Section 3.3: aggregate demand shocks generate small and transitory falls in productivity growth, while joint financial and aggregate demand shocks generate larger and persistent falls.

Figure 6: An example for a simulation run



[32]Indeed, there may be industry characteristics which affect the size distribution and also have a direct effect on productivity growth. In my model, this is the case for the parameter α, the probability of a firm to receive a radical innovation opportunity. These characteristics may blur the relationship suggested by Prediction 3 if they vary across industries.

[33]I did not specify the laws of motion for aggregate demand and financing conditions before, as they were irrelevant. Firms' R&D policy functions do not depend on transition probabilities for these processes, and for the balanced growth path and the impulse response analysis, I imposed a sequence of shocks.

I use these simulations to compare two industries which are identical expect for their value of the parameter ψ. To do so, I draw 500 different runs of aggregate shocks and calculate for each run the average and the standard deviation of productivity and output growth over the 100 periods of the simulation for both industries. Table 1 reports the average of both indicators over the 500 runs. It indicates that both the average and the standard deviation of productivity growth rates are higher in the high ψ industry, and that this industry has also on average a more left-skewed size distribution. The average and the standard deviation of output growth are positively correlated as well, as they are mainly driven by productivity. However, volatility differences are smaller, as output volatility is also driven by aggregate demand, which is the same for both industries.

Table 1: Simulation results

		$\psi = 0.1$	$\psi = 0.2$
Average production share of small firms		20.3%	54.6%
Productivity	Average growth rate	5.5%	8.2%
	Standard deviation	0.2%	0.6%
Output	Average growth rate	5.5%	8.2%
	Standard deviation	2.38%	2.41%

Notes: Small firms are defined as firms with two goods or less. Their production share in period t equals $m_{1,t} + 2m_{2,t}$. Each simulation run had 100 periods. Of these, there were on average 11.2 periods with negative demand shocks, and 2.9 periods with financial shocks. All parameter values other than ψ are the same as those used previously, and given in Table 7 in Appendix A.5.

3.5 Discussion of key assumptions

Before concluding on the model, it is useful to shortly discuss the role of some assumptions that have not been highlighted so far.

Several simplifying assumptions exclude aspects which are beyond the scope of my analysis. For instance, I consider a partial equilibrium model, and therefore an exogenous aggregate demand process, in order to avoid feedback effects from wages and dividends to aggregate demand. Such feedback effects may lead to the clustering of innovations (as has been pointed out by Shleifer (1986)), but they do not create differences between small and large firms. Likewise, I made assumptions eliminating precautionary effects due to the anticipation of potentially binding financial constraints in the future. This does not imply that these effects are negligible in practice.[34] However, as they do not affect the qualitative predictions of my model, I decide to abstract from them in order to get analytical solutions.

Finally, two assumptions play some role in the persistent rightward shift of the firm size distribution after a financial shock, which is key for my model's aggregate predictions.

[34] Pérez (2012) and Khan and Thomas (2013) show how they may affect investment dynamics in business cycle models.

First, the mass of potential entrants is a linear function of the mass of incumbents (see Equation (2)). Thus, the fall in the mass of incumbents after the shock also reduces the mass of potential entrants. This reinforces the persistence of the rightward shift of the size distribution, but it is not crucial. Indeed, the rightward shift in the size distribution could only be undone if entry would exceed its balanced growth path level in the aftermath of the shock.[35] However, there are no reasons for such an overshooting in my model, not even if the mass of potential entrants were pinned down by a free entry condition.[36]

Second, an important implicit assumption preventing entry from overshooting is that radical innovations cannot be stored: a firm that foregoes a radical innovation in period t cannot realise the same radical innovation in period $t+1$. This assumption can be interpreted as reflecting obstacles to postponing innovation.[37] However, even if it were possible to store radical innovations, the persistent effect of the shock would not disappear. Indeed, the size distribution would still be shifted to the right at least in the period immediately after the shock, lowering the number of radical innovations created in that period. Moreover, it is unlikely that all delayed innovations could actually be implemented, as there may be overlaps between them and the new innovations created in period $t+1$. For example, suppose a firm was prevented from increasing the frontier productivity of some good j from $a(j)$ to $\gamma a(j)$ in period t. In period $t+1$, another firm may be able to do the same, and as only one firm can actually innovate, the delayed firm may not be able to expand after all. Thus, allowing for the postponement of innovation would not eliminate my model's persistence channel.

Summing up, my model shows that firm-level heterogeneity matters for the analysis of the long-run implications of R&D fluctuations. Under the assumption that small firms' innovation capacity is relatively higher than the one of large firms, the model generates three main predictions. First, small firms' R&D react more to adverse shocks (Prediction 1). This explains that a financial shock, which triggers a rightward shift in the firm size distribution, persistently depresses productivity growth (Prediction 2). It also suggests that all else equal, industries with a more left-skewed firm size distribution have both higher productivity growth and volatility (Prediction 3). The remainder of the paper analyses the empirical evidence for these predictions.

[35] Thus, all results are qualitatively unchanged if I assume that the absolute mass of potential entrants is fixed (that is, if I replace Equation (2) with $m_{0,t} = \psi$).

[36] After a financial shock (and as long as the latter did not affect demand expectations), potential entrants would face the same problem than on the balanced growth path, with one exception: the mass of incumbents would be lower and therefore, the destruction rate would be higher for every given mass of entrants. This is due to the increase in the mass of non-contestable goods triggered by the shock. Thus, equilibrium entry would actually be lower than its balanced growth path level.
A free entry condition would, however, change the impulse responses to an aggregate demand shock. As the shock lowers the value of entry, it would now lead to a fall in entry and a persistent rightward shift in the firm size distribution, generating a persistent fall in productivity growth.

[37] For example, small firms may lose their skilled employees if they are prevented from realising their innovation opportunities, and it may be difficult to hire them back later.

4 Empirical evidence on the model's predictions

4.1 Firm-level differences in the sensitivity of R&D to aggregate shocks

Prediction 1 states that small firms reduce R&D investment more than large firms after a negative aggregate shock. This prediction is in line with several empirical studies comparing small and large firms' reactions to aggregate shocks. Gertler and Gilchrist (1994) show that small manufacturing firms' inventories fall after a negative monetary policy shock, while those of large firms increase. Fort et al. (2013) extend this evidence by taking into account firm age.[38] They show that the job creation rate of young and small firms falls more than the one of old and large firms during economic downturns. This differential is particularly large for the 2007-2009 Great Recession.[39] Many studies also document a large fall in entry and firm creation during the Great Recession (OECD (2012), Sedlacek and Sterk (2013), Klapper et al. (2014)), suggesting that potential entrants were also strongly affected.

However, only a small number of studies have explicitly considered R&D cyclicality by firm size.[40] In the remainder of this section, I contribute to fill this gap by using a German firm-level dataset to compare the response of small and large firms' R&D to the Great Recession, which can be considered as a joint aggregate demand and financial shock.

My dataset comes from the Mannheim Innovation Panel (MIP), an annual survey carried out by the Centre for European Economic Research (ZEW). The survey targets a representative sample of German firms with 5 employees or more, in a broad range of innovating sectors. It asks firms about R&D, total innovation spending (including R&D, implementation and marketing expenses) and some other variables such as sales or employment. The dataset is an unbalanced panel covering the period 1999-2009. A more complete description of its characteristics can be found in Appendix B.1.

In my model, all firms are potential innovators. In reality, however, many firms never spend on R&D or introduce innovations. Thus, I restrict the sample to firms which have at least one observation with non-zero R&D,[41] to focus on the population of innovating firms.

Table 2 shows that my model's Prediction 1 holds during the Great Recession in Germany. In the population of innovating firms with non-trivial R&D[42], the median small firm (with 50 employees or less) increased R&D

[38] In my model, I do not consider firm age explicitly, because it is highly correlated with firm size.

[39] Krueger and Charnes (2011) and Siemer (2013) also claim that small firms' employment suffered disproportionately during the Great Recession. Moscarini and Postel-Vinay (2012) take a dissenting stand.

[40] Paunov (2012) shows that in a panel of firms from eight Latin American countries, young firms were more likely to abandon innovation investment during the Great Recession. She does not find an independent role for size. Hall (2011), on the other hand, presents preliminary evidence for small firms' R&D investment falling more during the same period in the United States. Finally, Aghion et al. (2012) show that R&D investment reacts more to negative sales shocks in financially constrained firms. However, even though most conventional measures suggest that constrained firms are on average smaller than unconstrained ones (Farre-Mensa and Ljungqvist (2013)), Aghion et al.'s proxy for financial constraints is uncorrelated with firm size.

[41] This is common practice in firm-level R&D studies (see, for instance, Aghion et al. (2012)).

[42] I exclude in Table 2 changes for firms which spend in both years less than 10.000€ on R&D, in order not to overweight

less in 2008 and reduced it more in 2009 than the median large firm (with more than 500 employees). This is remarkable, as the table also shows that small firms were a priori hit less intensely by the crisis than large ones: their sales increased more in 2008 and fell less in 2009, probably due to their smaller exposition to the collapse in world trade.

Table 2: R&D and sales fluctuations by firm size during the Great Recession

	2008			**2009**		
Median rate of change	**R&D**	**Sales**	**Obs.**	**R&D**	**Sales**	**Obs.**
Small (50 or less emp.)	0.00	6.45	283	-3.68	-3.08	363
Large (more than 500 emp.)	4.88	1.90	121	-2.13	-14.06	103

Notes: Statistics are computed on the sample of innovating firms which have observations for at least two consecutive years. They exclude firms with trivial R&D investment (smaller than 10000€ in both years). Rates of change for a variable x are computed as $100\frac{x_t - x_{t-1}}{0.5(x_t + x_{t-1})}$, following Davis et al. (1998). Employment refers to the first year of observation (i.e., 2007 for the rate of change between 2007 and 2008).

Conditioning on the sign of the sales change (to compare firms affected with similar intensity by the crisis) makes differences between small and large firms even more salient. Table 3 shows that among firms which saw their sales fall during the Great Recession, small firms cut R&D investment considerably more than large ones (especially in 2009). Among firms with increasing sales, small and large firms increased R&D more or less proportionally to their sales.[43] When considering means instead of medians, or total innovation spending instead of R&D investment, results are similar. The corresponding tables are shown in Appendix B.2.

Table 3: R&D fluctuations conditional on the sign of sales changes

	2008			**2009**		
Median rate of change	**R&D**	**Sales**	**Obs.**	**R&D**	**Sales**	**Obs.**
Firms with decreasing sales						
Small (50 or less emp.)	-8.00	-10.83	87	-36.07	-18.70	201
Large (more than 500 emp.)	0.00	-6.23	45	-7.47	-21.13	76
Firms with increasing sales						
Small (50 or less emp.)	14.59	11.76	196	13.61	10.64	162
Large (more than 500 emp.)	10.97	7.84	76	10.52	6.81	27

Notes: See notes to Table 2. Firms with unchanged sales are included in the increasing sales category.

Tables 2 and 3 suggest that small firms reacted more than large ones to the Great Recession. This is especially

changes in trivial amounts. Results are unchanged with a threshold of 20.000€.

[43] Tables 2 and 3 also confirm the procyclicality of R&D indicated by aggregate data: firms in general increase R&D if sales increase and decrease R&D if sales fall.

true when comparing the evolution of R&D to the one of sales, which can be seen as a proxy for how intensely the Great Recession affected a firm.[44] Thus, a gap between the R&D intensity (the ratio of R&D to sales) of small and large firms opened up, in disfavour of small firms. However, the preceding tables do not show whether this gap is statistically significant.

I therefore analyse a series of fixed effect regressions, exploiting the panel structure of my dataset. The dependent variable in the regressions is firms' R&D intensity, and I estimate

$$\begin{aligned}\frac{\mathrm{RD}_{it}}{\mathrm{Sales}_{it}} = \quad & \alpha_i + \gamma \ln\left(\mathrm{Employment}_{it}\right) + \textstyle\sum_{\tau=1999}^{2009} \beta_\tau D_t^\tau + \beta_{SE}\mathrm{SE}_{it} + \textstyle\sum_{\tau=1999}^{2008} \beta_{SE,\tau}\mathrm{SE}_{it}D_t^\tau \\ & + \beta_{ME}\mathrm{ME}_{it} + \textstyle\sum_{\tau=1999}^{2008} \beta_{ME,\tau}\mathrm{ME}_{it}D_t^\tau + \varepsilon_{it}\end{aligned}, \qquad (20)$$

where α_i is a firm-level fixed effect which controls for time-invariant factors affecting R&D intensity. SE_{it} is a dummy for firm i being small, that is, having 50 employees or less in the last period of observation before t, while ME_{it} is a dummy for medium-size firms, with between 51 and 500 employees.[45] Large firms, with more than 500 employees, are the omitted category, so all coefficients must be interpreted with respect to them. D_t^τ are time dummies for the year τ. I winsorize outliers for R&D intensity (5% of the highest non-zero values for every year) and cluster standard errors at the firm and at the industry-year level.

Results are shown in the left column of Table 4. To interpret them, note that coefficients are identified by within-firm variation. Equation (20) is specified such that the interaction between the year dummy for 2009 and the small firm dummy is the omitted category. Therefore, the negative and significant estimate for β_{SE} means that, with respect to their within-firm average, small firms had a lower R&D intensity than large ones in 2009. This confirms that the gap in the relative R&D intensity of small and large firms during the Great Recession is statistically significant. The positive and significant estimates for the coefficients of the other interactions between year and small firm dummies indicate that the gap between small and large firms was significantly smaller in those years (around 0.9 percentage points).[46] Estimates for medium-size firms (not shown) have the same signs than those for small firms, but are in general insignificant.

Using R&D intensity as a dependent variable controls for industry-level differences in sales shocks, but does not control for potential industry-level differences in the elasticity of R&D with respect to sales.[47] Therefore, as a robustness check, I test whether findings hold in a subset of "research-intensive manufacturing industries",[48] which account for almost two thirds of German R&D spending. Results for this subsample are

[44] In the model, all firms' sales are equally affected by an aggregate shock. In reality, however, many factors which are not included in the model may explain that firms were more or less affected by the Great Recession.

[45] For a firm's first period of observation, the value of the dummy is determined with respect to present employment.

[46] The gap is also economically significant, as the R&D intensity of a firm rarely exceeds 5%. Note, however, that these results just establish a stylized fact. They do not prove that the Great Recession is the reason for the gap.

[47] This is the reason for which I exclude R&D Services (NACE Code 73) from the baseline results presented in Table 4. This industry is composed almost exclusively by small firms, which see their R&D intensities fall substantially in 2008 and 2009. This is in line with my argument, but nevertheless, it would be suspicious if results were driven by this, as it could be due to a particularity of this industry as well as to a greater crisis reaction of small firms.

[48] Under this header, the ZEW groups Chemicals, Machinery, Electrical and Optical Equipment and Transport Equipment.

shown in the right column of Table 4, and are in line with the full sample. I also estimated (20) with a full set of industry-year dummies instead of only year dummies. Results (shown in Appendix B.2) are similar to the ones shown in Table 4.

Table 4: Fixed effect regression results

Dependent variable: R&D intensity

Sample	All firms	Res.-int. manuf.
SE	$-1.10\,(0.45)^{**}$	$-2.49\,(0.92)^{***}$
SED^{1999}	$1.36\,(0.52)^{***}$	$2.17\,(0.94)^{**}$
SED^{2000}	$1.05\,(0.47)^{**}$	$3.29\,(0.82)^{***}$
SED^{2002}	$2.46\,(0.48)^{***}$	$4.40\,(0.82)^{***}$
SED^{2003}	$1.39\,(0.47)^{***}$	$3.12\,(0.83)^{***}$
SED^{2004}	$2.25\,(0.44)^{***}$	$3.21\,(0.91)^{***}$
SED^{2005}	$1.16\,(0.42)^{***}$	$2.00\,(0.78)^{**}$
SED^{2006}	$0.93\,(0.39)^{**}$	$2.01\,(0.78)^{***}$
SED^{2007}	$0.92\,(0.38)^{**}$	$2.50\,(0.81)^{***}$
SED^{2008}	$0.84\,(0.43)^{**}$	$1.30\,(0.65)^{**}$
Observations	15206	5729

Notes: *** significant at 1%, ** significant at 5%, * significant at 10%. Estimates for the coefficients on year dummies, medium-size firms and ln (Employment) are not shown. The R&D Services industry (NACE Code 73) is left out. There are no estimates for 2001, as the survey for this year did not contain a question on R&D. Cluster-robust standard errors are given in parentheses. There are 4234 clusters at the firm and 343 at the industry-year level in the regression for all firms, and 1681 firm and 80 industry-year clusters in the research-intensive manufacturing sample (industries are classified according to the two-digit NACE Rev. 1.1 classification).

I consider several other robustness checks. I specify the dependent variable in natural logarithms, omit firms which are observed only for two years or consider innovation spending intensity instead of R&D intensity as the dependent variable. In each case, results are qualitatively unchanged.[49]

In sum, small German firms on average reduced R&D more than large ones during the Great Recession. As a result, at the high point of the recession in 2009, the gap between large and small firms R&D intensity (in disfavour of small firms) was higher than in any year of the preceding decade.

The empirical support for Prediction 1, together with the evidence for the higher relative innovation capacity of small firms discussed in Section 2.4, makes my model's aggregate predictions more plausible. However, aggregate predictions can also be examined directly. I turn to this in the two remaining sections.

[49]Results for innovation spending intensity are shown in Appendix B.2, all other results are available upon request.

4.2 The impact of a financial shock on productivity growth

My model predicts that a financial shock has a persistent negative effect on productivity growth, arising through a persistent rightward shift in the size distribution of innovating firms (Prediction 2).

This prediction seems consistent with the experience of OECD countries after the Great Recession. In the United States, the observation that productivity growth in the recovery appears to be depressed with respect to earlier levels has recently received a lot of attention.[50] However, it is unclear whether this is due to the financial shocks in the Great Recession. Indeed, Fernald (2014) argues that the slowdown started even earlier and is due to the fading out of the IT-driven productivity improvement wave of the 1990s. In other countries, the case for a slowdown induced by financial disruptions is stronger. Ball (2014) shows the fall in the growth rate of potential output after the Great Recession (driven to an important extent by productivity growth) was relatively small in the United States compared to other OECD countries. In the periphery of the Eurozone (Spain, Portugal, Greece, Ireland), where disruptions in the financial system have arguably been more severe and more prolonged, the fall in potential output growth was substantially larger.[51]

However, even if productivity growth has slowed down in some countries, is this due to a rightward shift in the size distribution of innovating firms? It is difficult to assess this claim in the data, as there is little information on the evolution of the size distribution of innovating firms over time. Existing data for Germany and the United States is consistent with a (small) rightward shift. In Germany, the share of small firms (with less than 50 employees) in the population of manufacturing firms reporting continuous or occasional R&D activities fell from 68.4% in 2008 to 66.3% in 2010. In services, it fell from 83.3% to 81.9%.[52] In the United States, the share of firms with less than 500 employees in aggregate R&D fell from 21.2% in 2008 to 19.3% in 2010.[53] These observations are consistent with my model, even though they of course do not prove a causal effect of the rightward shift on the productivity slowdown.

4.3 Growth, volatility and the size distribution of innovating firms

My model predicts that, all else equal, there should be a positive correlation between the innovation share of small firms, the average and the standard deviation of productivity growth between industries (Prediction 3).

[50] See for instance the cover story of *The Economist*, July 19th, 2014.

[51] According to Ball's estimates, potential growth fell by 0.34 percentage points (pp) in the United States, but by 2.64 pp in Spain, 1.34 pp in Portugal, 4.11 pp in Greece and 4.82 pp in Ireland. The smaller long-run impact of the crisis in the United States is compatible with my model, if one assumes that entry and creative destruction are structurally higher in the United States than in the Eurozone periphery. Numerical simulations indeed predict that productivity growth returns faster to its balanced growth path after a financial shock if the relative mass of potential entrants ψ is higher, because the rightward shift in the size distribution is corrected more quickly.

[52] These figures are computed from ZEW estimates based on the MIP (Rammer et al. (2010, 2012), Page 16).

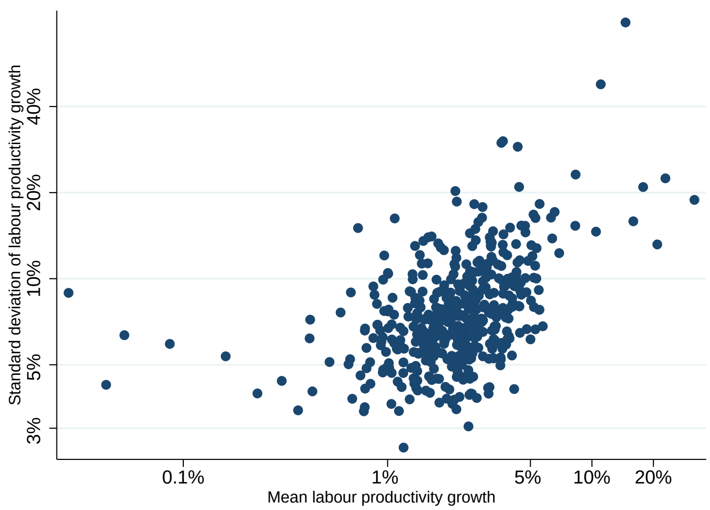
[53] The figures are computed from the data tables of the Business Research and Development and Innovation Survey (BRDIS), carried out by the National Science Foundation (NSF) and available at www.nsf.gov/statistics/srvyindustry. It is unfortunately more difficult to compare this data to pre-crisis years such as 2006 or 2007, as the BRDIS survey was introduced only in 2008 and several changes with respect to its predecessor may have created discontinuities.

I show in this section that within the United States manufacturing sector (whose industries should have experienced at least similar financial and aggregate demand shocks), the average and the standard deviation of productivity growth are indeed positively correlated across industries. The scarce data on the size distribution of innovating firms makes it hard to test whether this new stylized fact[54] is linked to the innovation share of small firms, as predicted by my model. However, the existing evidence is consistent with this hypothesis.

4.3.1 Productivity growth and volatility across industries

To assess the correlation between productivity growth and volatility across industries, I use the NBER-CES Manufacturing database (Becker et al. (2013)). Following Acemoglu et al. (2014), I measure an industry's productivity as real shipments per employee (that is, labour productivity).[55]

Figure 7: Productivity growth and volatility across US manufacturing industries, 1972-2009



Note: Both axes have a logarithmic scale.

Figure 7 plots, on a logarithmic scale, the average growth rate of labour productivity between 1972 and 2009 against the standard deviation of productivity growth rates, measured at the level of six-digit NAICS industries. It indicates a strong positive correlation between both variables.

The positive correlation between growth and volatility is not affected by the introduction of control variables. In particular, I estimate the model

$$\ln sd_k^{LP} = \beta_0 + \beta_1 \ln g_k^{LP} + \beta_2 \text{Size}_k + \beta_3 \text{Capital Share}_k + \beta_4 sd_k^{CU} + \varepsilon_k, \tag{21}$$

[54] Imbs (2007) shows that there is a positive correlation between the average and the standard deviation of output growth at the industry level. However, this does not imply a priori that the same correlation holds for productivity.

[55] Further details on the variables and measures used are given in Appendix B.3.

where sd_k^{LP} stands for the standard deviation of labour productivity growth rates in industry k and g_k^{LP} for their average. I consider three control variables: industry size (measured by the natural logarithm of average value added over the period), the capital share (measured by the average capital share in the industry's production function[56]) and the standard deviation of capacity utilisation (sd_k^{CU}).[57] Table 5 shows that larger industries have lower volatility, while the other two control variables are insignificant. The correlation between productivity growth and volatility remains positive and strongly significant.

Table 5: Productivity Growth and Volatility: Regression Evidence

Dependent variable: ln (Standard deviation of productivity growth)

ln (Average productivity growth)	0.268***	0.282***
	(0.040)	(0.039)
ln (Average value added)		−0.162***
		(0.017)
St. Dev. Capacity Utilisation		0.567
		(0.637)
Capital share		0.141
		(0.206)
Constant	−1.552***	−0.386*
	(0.154)	(0.219)
R^2	0.206	0.362
Observations	457	457

Notes: Productivity is measured by real shipments per employee. Average value added of the industry is given in millions of dollars. Robust standard errors are given in parentheses.

Results presented in this section do not change if I consider instead of labour productivity the measure of total factor productivity (TFP) provided in the NBER-CES database (calculated as a Solow residual from a neoclassical production function). They are also robust to omitting petrol industries, weighting observations by industry size or using the average yearly growth rate of productivity (i.e., the annualized equivalent of the growth rate of productivity from 1972 to 2009) rather than the average of annual growth rates.[58]

In sum, the results of this section suggest a strong positive correlation between productivity growth and

[56] See Becker et al. (2013) for details on the measurement of the capital share.

[57] The labour productivity measure used may indeed overstate the volatility of productivity growth in industries with a volatile capacity utilisation rate. Data on capital utilisation is provided monthly by the Federal Reserve (accessible at http://www.federalreserve.gov/RELEASES/G17/caputl.htm) for 22 manufacturing sectors. I take averages of these observations over time for every sector, and assign to every industry in the database the average of the corresponding sector.

[58] These results are available upon request.

volatility at the industry level within US manufacturing.[59] I now briefly analyse whether this correlation can be linked to differences in the size distribution of innovating firms.

4.3.2 The role of the size distribution of innovating firms

As mentioned in Section 4.2, data on the size distribution of innovating firms is scarce, especially at the industry level. Table 6 shows some partial evidence for the US manufacturing sector. It indicates the share of Small and Medium Enterprises (SMEs) in industry R&D (which may be interpreted as a proxy for small firms' share of innovation) as well as average productivity growth and volatility for the three 3-digit NAICS industries which contribute most to aggregate R&D.

Table 6: SME's share in R&D, productivity growth and volatility in selected industries

		SME share in R&D			**Productivity (1972-2009)**	
	NAICS code	**1997**	**2004**	**2010**	**Avg. Growth**	**Volatility**
Computer and Electronic Products	334	7%	13%	13%	10.6%	8.4%
Chemicals	325	3%	8%	10%	2.4%	4.4%
Transportation Equipment	336	0%	2%	5%	2.5%	5.0%

Notes: SME shares in R&D are measured as shares of domestic R&D financed with company or other non-federal funds carried out by firms with 500 employees or less (except for 2010, where other non-federal funds are excluded). Data comes from the National Science Foundation (www.nsf.gov). For 1997, the share for the Computer and Electronic Products industry is an average between the industries Office, computing, and accounting machines and Electrical Equipment. Productivity (real shipments per employee) growth and volatility are calculated using the NBER-CES Manufacturing database.

The table shows that the computer and electronic products industry, which has the largest share of R&D carried out by SMEs, also has the highest average and standard deviation of productivity growth. Traditional manufacturing industries such as the car industry (Transportation Equipment) and the chemical industry, where R&D is dominated by large firms, have grown slower and fluctuated less.

In high-tech service industries, SMEs shares in R&D are often even much higher than in computer manufacturing (for example, they exceed 50% in Computer Systems Design or in R&D services). However, there is in general no productivity data for these industries (even though they have indeed had high and volatile R&D growth rates over the last decades, in line with my model).

In sum, my model provides a mechanism that can explain the positive correlation between productivity growth and volatility at the industry level: some exogenous fundamentals make the size distribution of innovating firms more left-skewed in some industries than in others, resulting in higher average productivity growth and

[59] However, results should be analysed with caution because of the limits of the productivity data used: both TFP and labour productivity are revenue-based concepts and therefore confound the effect of markups and physical productivity.

higher volatility. This mechanism is consistent with the available data, but its quantitative relevance ought to be examined with better industry-level data once it becomes available.

5 Conclusion

The model developed in this paper shows that taking into account firm-level heterogeneity provides new insights on the long-run impact of R&D fluctuations. Indeed, while small firms have a relatively higher innovation capacity than large firms, they also reduce R&D more after a negative aggregate shock. This implies that financial shocks shift the size distribution of innovating firms persistently to the right and thereby persistently lower productivity growth. Furthermore, the share of small firms in innovation, productivity growth and volatility are positively correlated at the industry level. The empirical evidence is consistent with the model's predictions.

Exploring the quantitative implications of my model is beyond the scope of this paper. However, after relaxing a series of assumptions made to preserve analytical tractability, my model could be calibrated to firm-level data in order to provide an estimate for the productivity loss triggered by a financial crisis. Such a quantitative model could also be used to study the effect of government policies, such as countercyclical R&D subsidies targeted at small firms.

Finally, the differences in small and large firms' R&D behaviour highlighted in this paper may also provide a new perspective on the effect of permanent aggregate shocks, such as globalisation. Globalisation has increased the potential market size for small, innovative start-up firms, but it also appears to have further increased the weight of the very largest firms in the world economy. Exploring the implications of these changes for the size distribution of innovating firms and aggregate productivity growth is a promising topic for further research.

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A Theoretical Appendix

A.1 The firm's dynamic programming problem

The firm's problem An incumbent firm with n_t goods ($n_t \geq 1$) and cash holdings c_t solves

$$\max_{(\mathbf{1}_{R,s},\mathbf{1}_{I,s})_{s\geq t}} E_t\left(\sum_{s=t}^{+\infty} \mathbf{1}_{E,s}\tilde{c}_s\right),$$

where $\mathbf{1}_{E,s}$ is an indicator function for the firm still existing in period s and $\tilde{c}_s$ is its cash flow in that period. Cash flows depend on R&D as described in the main text. For instance, if the firm realises a radical innovation opportunity in period s, this yields cash flows of $-f_R$ in period s and $\left(1-\frac{1}{\gamma}\right)S_{s+1}$ in period $s+1$. Maximization faces two constraints. First, in periods with crisis financing conditions, Equation (5) must hold. This makes cash holdings a state variable. Second, the firm can only collect cash flows only as long as it is exists ($\mathbf{1}_{E,s}=1 \Leftrightarrow n_s \geq 1$). The number of goods produced by the firm follows the stochastic process

$$n_{s+1} = \begin{cases} n_s - 1 & \text{with probability } d_s\left(1-\alpha\mathbf{1}_{R,s}\right) \\ n_s + 1 & \text{with probability } \alpha\mathbf{1}_{R,s}\left(1-d_s\right) \\ n_s & \text{else} \end{cases} \quad \text{if } n_s \geq 1.$$

Once a firm falls to zero goods, it exits forever. Potential entrants in period t solve the same problem, but they have $\mathbf{1}_{E,t}=1$ (they exist in the beginning of period t even though they produce no good). If they receive a radical innovation opportunity and decide to pay the radical R&D cost, they enter and have one good in the beginning of the next period. Otherwise, they stay out forever.

A basic property of the value function I define V as the value of an incumbent's problem in period t, after earning current profits and before learning whether it receives a radical innovation opportunity. V depends on the endogenous state variables n_t and c_t, but also on aggregate demand next period, on current financing conditions and on the anticipated path of future destruction rates $(d_s)_{s\geq t}$. This path depends on the joint distribution of goods and cash across firms (which determine the mass of constrained firms if a financial shock hits), so the problem must in principle be solved with the Krusell and Smith (1998) algorithm. However, the parameter restrictions made in the main text simplify the problem substantially.

First, notice that the value function V is non-decreasing in n_t and in c_t. Indeed, a firm with n_t goods can mimic any strategy of a firm with a lesser number of goods (while the reverse is not possible) and therefore must achieve on expectation at least the value of that firm.[60] The same observation holds for cash levels.

Policy functions To simplify notation, I regroup exogenous state variables in the (infinite-dimensional) vector $\mathbf{\Phi}_t$ and denote by $NPV_{R,t}$ and $NPV_{I,t}(n_t)$ the NPV delivered by radical or incremental innovation paid for in period t for a firm with n_t goods.[61] Consider first a period t in which financing conditions are normal. The Bellman equation for an incumbent's problem is

$$V(n_t, c_t, \mathbf{\Phi}_t) = (1-\alpha)\max(V_1, V_3) + \alpha \max(V_1, V_2, V_3, V_4),$$

where V_1 to V_4 are the values associated to the firm's choices in period t. V_1 is the expected value of the firm when doing neither radical nor incremental R&D:

$$V_1 = d_t E_t(V(n_t - 1, c_t, \mathbf{\Phi}_{t+1})) + (1-d_t) E_t(V(n_t, c_t, \mathbf{\Phi}_{t+1})).$$

V_2 is the expected value when doing radical, but no incremental R&D:

$$V_2 = NPV_{R,t} + d_t E_t(V(n_t, c_t + NPV_{R,t}, \mathbf{\Phi}_{t+1})) + (1-d_t) E_t(V(n_t + 1, c_t + NPV_{R,t}, \mathbf{\Phi}_{t+1})).$$

V_3 is the expected value when doing incremental, but no radical R&D:

$$V_3 = NPV_{I,t}(n_t) + d_t E_t(V(n_t - 1, c_t + NPV_{I,t}(n_t), \mathbf{\Phi}_{t+1})) + (1-d_t) E_t(V(n_t, c_t + NPV_{I,t}(n_t), \mathbf{\Phi}_{t+1})).$$

and V_4 is the expected value when doing both radical and incremental R&D:

$$\begin{aligned} V_4 = \quad & NPV_{R,t} + NPV_{I,t}(n_t) + d_t E_t(V(n_t, c_t + NPV_{R,t} + NPV_{I,t}(n_t), \mathbf{\Phi}_{t+1})) \\ & + (1-d_t) E_t(V(n_t + 1, c_t + NPV_{R,t} + NPV_{I,t}(n_t), \mathbf{\Phi}_{t+1})). \end{aligned}$$

[60] If the optimal strategy of the smaller firm involves incremental R&D, the value of the larger firm is actually strictly higher than that of the smaller one, as it earns more profits from incremental innovation.

[61] That is, $NPV_{R,t} = \left(1 - \frac{1}{\gamma}\right) S_{t+1} - f_R$ and $NPV_{I,t}(n_t) = (n_t - 1)\left(1 - \frac{1}{\delta}\right) S_{t+1} - f_I$.

With probability $1-\alpha$, the firm receives no radical innovation opportunity and therefore only has the choice between V_1 and V_3. Clearly, V_3 is larger than V_1 if and only if $NPV_{I,t}(n_t) > 0$ (which is true if n_t is larger or equal to the threshold size $n^*_{I,t}$ defined in the main text). In this case, incremental innovation has a positive NPV and increases cash next period for sure. Both effects have some non-negative value for the firm (the second because V is non-decreasing in cash), so choosing incremental R&D is better than not doing it. Likewise, $n_t \geq n^*_{I,t}$ also implies $V_4 > V_2$. As radical innovation always delivers a positive NPV and increases both the cash holdings and the number of goods produced next period, it is always true that $V_2 > V_1$ and $V_4 > V_3$. Thus, in a period with normal financing conditions, all firms do radical R&D if they receive a radical innovation opportunity,[62] and they do incremental R&D if and only if $n_t \geq n^*_{I,t}$.

In a period with crisis financing conditions, the previous analysis remains valid, meaning that firms' desired R&D decisions do not change. Thus, actual R&D decisions remain the same as under normal financing conditions as long as the firms can finance them with their cash holdings. Firms that would want to do both radical and incremental R&D, but only have enough cash to do one of the two, prefer to maintain radical R&D because it delivers both a higher NPV (because of Restriction 3) and an increase in expected firm size. Thus, firms do radical R&D if and only if they get a radical innovation opportunity and their cash holdings exceed f_R, and do incremental R&D if their subsequent cash holdings exceed f_I.

A.2 Proof of Equation (12): the law of motion of the aggregate price level

Replacing the pricing Equation (8) into the definition of the aggregate price level in (7) yields $P_t = \exp\left(\int_0^1 \ln \frac{w_t}{a_{t-1}(j)} dj\right)$. From period t to period $t+1$, the price of a mass R_{t-1} of goods is multiplied by $\frac{w_{t+1}}{\gamma w_t}$, the price of a mass I_{t-1} by $\frac{w_{t+1}}{\delta w_t}$ and the price of the remainder by $\frac{w_{t+1}}{w_t}$. Therefore,

$$P_{t+1} = \frac{w_{t+1}}{w_t} \exp\left(\int_0^1 \ln p_t(j) + R_{t-1} \ln\left(\frac{1}{\gamma}\right) + I_{t-1} \ln\left(\frac{1}{\delta}\right) dj\right) = P_t \frac{w_{t+1}}{w_t} \exp\left(-\left(R_{t-1} \ln \gamma + I_{t-1} \ln \delta\right)\right).$$

A.3 The invariant firm size distribution

On the balanced growth path, for every $n \geq 2$, the law of motion in (15) can be rewritten as

$$m_{n+1} = \left(1 + \frac{\alpha(1-d)}{(1-\alpha)d}\right) m_n - \frac{\alpha(1-d)}{(1-\alpha)d} m_{n-1}.$$

The characteristic equation of this sequence has roots 1 and $\frac{\alpha(1-d)}{(1-\alpha)d}$. Therefore, there exist two real numbers C_1 and C_2 such that

[62] This reasoning also applies to potential entrants, which cannot do incremental R&D and whose outside option when not taking the radical innovation opportunity (staying out forever) has value 0.

$$\forall n \geq 1, \quad m_n = C_1 + C_2 \left(\frac{\alpha (1-d)}{(1-\alpha) d} \right)^{n-1}.$$

As $\sum_{n=1}^{+\infty} m_n$ must be finite, $C_1 = 0$. This implies $C_2 = \frac{\alpha}{(1-\alpha)d} m_0$. Furthermore, note $d = \alpha (1+\psi)$ to get

$$\forall n \geq 1, \quad m_n = \frac{m_0}{(1-\alpha)(1+\psi)} \left(\frac{1-\alpha(1+\psi)}{(1-\alpha)(1+\psi)} \right)^{n-1}.$$

m_0 is pinned down by the condition that all goods must be produced, that is, $\sum_{n=1}^{+\infty} n m_n = 1$.

An increase in ψ shifts the invariant distribution to the left, as claimed in Footnote 27.

$$\forall n \geq 1, \quad \frac{\partial m_n}{\partial \psi} = \psi \frac{(1-\alpha(1+\psi))^{n-2}}{((1-\alpha)(1+\psi))^{n+1}} \left(2(1-\alpha) - \frac{(n+1)\psi}{1+\psi} \right).$$

This has the sign of $2(1-\alpha) - \frac{(n+1)\psi}{1+\psi}$. It is easy to show that $\frac{\partial m_1}{\partial \psi} > 0$, using the condition $\alpha(1+\psi) < 1$. For all other n, the sign of the derivative is indeterminate. However, the term defining the derivative's sign is clearly decreasing in n, and $\lim_{n \to +\infty} 2(1-\alpha) - \frac{(n+1)\psi}{1+\psi} = -\infty$. This implies that there is a threshold size $n^* \geq 2$ at which the sign of $\frac{\partial m_n}{\partial \psi}$ changes from positive to negative.

A.4 Tracking the fraction of unconstrained firms

I need to compute for every period with crisis financing conditions the fractions of firms of each size class which have cash holdings exceeding f_I, f_R or $f_I + f_R$. These fractions are key for determining aggregate innovation masses (Equations (10) and (11)) and the evolution of the firm size distribution (Equation (15)). I calculate these fractions by tracking the joint distribution of size and cash for firms with cash holdings low enough to make them constrained under crisis financing conditions (that is, for firms with cash holdings lower than $f_I + f_R$).[63]

The evolution of the mass of firms which are in state (n_t, c_t) in period t (that is, which produce n_t goods and have cash holdings c_t after production) depends on the current state of financing conditions. When financing conditions are normal, four different cases must be considered:

1) A fraction $\alpha(1-d_t)$ of firms transitions to state $\left(n_t + 1, c_t + NPV_{R,t} + \mathbf{1}_{n_t \geq n^*_{I,t}} NPV_{I,t}(n_t)\right)$, where $\mathbf{1}_{n_t \geq n^*_{I,t}}$ is an indicator function for the firm producing $n^*_{I,t}$ goods or more. These firms gain one good by radical innovation, do not lose their contestable good and increase their cash holdings with the cash flow from radical innovation (and incremental innovation, if they are large enough to do incremental R&D).

2) A fraction αd_t transitions to state $\left(n_t, c_t + NPV_{R,t} + \mathbf{1}_{n_t \geq n^*_{I,t}} NPV_{I,t}(n_t)\right)$.

3) A fraction $(1-\alpha)(1-d_t)$ transitions to state $\left(n_t, c_t + \mathbf{1}_{n_t \geq n^*_{I,t}} NPV_{I,t}(n_t)\right)$.

[63] Recall that as cash holdings never fall, all firms which pass this threshold are forever unconstrained.

4) The remaining fraction $(1-\alpha) d_t$ transitions to state $\left(n_t - 1, c_t + \mathbf{1}_{n_t \geq n^*_{I,t}} NPV_{I,t}(n_t)\right)$, or exits if $n_t = 1$. Furthermore, a mass $\alpha m_{0,t}$ of entrants transition to state $(1, \tilde{c}_{R,t})$.

Under crisis financing conditions, there is no entry, and transitions of incumbent firms depend on current cash holdings. If $c_t < f_R$, firms cannot do radical innovation. Then, there are two cases for firms in state (n_t, c_t) in period t: a fraction $(1-d_t)$ transitions to state $\left(n_t, c_t + \mathbf{1}_{n_t \geq n^*_{I,t}} \mathbf{1}_{c_t \geq f_I} NPV_{I,t}(n_t)\right)$ and the remaining fraction d_t transitions to state $\left(n_t - 1, c_t + \mathbf{1}_{n_t \geq n^*_{I,t}} \mathbf{1}_{c_t \geq f_I} NPV_{I,t}(n_t)\right)$.

Finally, if $c_t \geq f_R$, firms can do radical innovation, and there are again four cases for firms in state (n_t, c_t) in period t: a fraction $\alpha(1-d_t)$ transitions to state $\left(n_t + 1, c_t + NPV_{R,t} + \mathbf{1}_{n_t \geq n^*_{I,t}} \mathbf{1}_{c_t \geq f_I + f_R} NPV_{I,t}(n_t)\right)$, a fraction αd_t transitions to state $\left(n_t, c_t + NPV_{R,t} + \mathbf{1}_{n_t \geq n^*_{I,t}} \mathbf{1}_{c_t \geq f_I + f_R} NPV_{I,t}(n_t)\right)$, a fraction $(1-\alpha)(1-d_t)$ transitions to state $\left(n_t, c_t + \mathbf{1}_{n_t \geq n^*_{I,t}} \mathbf{1}_{c_t \geq f_I} NPV_{I,t}(n_t)\right)$ and the remaining fraction $(1-\alpha) d_t$ transitions to state $\left(n_t - 1, c_t + \mathbf{1}_{n_t \geq n^*_{I,t}} \mathbf{1}_{c_t \geq f_I} NPV_{I,t}(n_t)\right)$, or exit if $n_t = 1$.

Using these transition processes, I can determine the joint distribution of size and cash holdings in period $t+1$ as a function of the one in period t. This allows me to calculate the fraction of firms of each size class that have less than any given level of cash holdings, and this directly yields the fractions $u^f_{n,t}$.

On the balanced growth path, I could start from an arbitrary distribution of potentially constrained firms and iterate the laws of motion described above until they converge. However, there is also a somewhat simpler method to compute the masses of potentially constrained firms, taking into account firm age.

I denote by $m_n(c, \mathfrak{a})$ the mass of firms with n goods, cash holdings c and age $\mathfrak{a}$. Firms of age 1 (last period's entrants) must necessarily have one good and cash holdings NPV_R. Therefore, $m_1(NPV_R, 1) = \alpha m_0$. From this, I deduce the masses of firms of age 2, using the same transition processes as before and taking into account that all firms of age 2 must have had age 1 last period. Iterating forward, I calculate the distribution of size and cash for every age $\mathfrak{a}$, and get the unconditional distribution by integrating over this last dimension.

A.5 Parameter values for simulations

For Figures 3 to 6, I assume $\psi = 0.2$. All other parameter values are given in Table 7.

Table 7: Parameter values

Parameter	Value	Parameter	Value
γ	1.16	f_R	0.1
δ	1.03	f_I	0.025
S_H	1	α	0.7
S_L	0.85	$L(w)$	$\sqrt{w}$

B Empirical Appendix

B.1 The MIP survey and my dataset

The ZEW sends the MIP survey every year to a representative panel of German firms with at least 5 employees in "innovating" industries.[64] The panel is updated every two years, with firms exiting the target population being replaced by new ones to maintain representativeness.

While all firms in the panel receive the survey every year, they unfortunately do not all answer it. The dataset is therefore a highly unbalanced panel, as firms may respond in some years but not in others. The selection bias arising from voluntary participation, however, does not appear to be very important. Non-respondent surveys conducted by telephone have shown that the share of innovating firms among responding firms is only slightly lower than in the entire sample (Peters (2009)).

In my paper, I consider the period 1999-2009. Earlier data, for 1992-1998, is available but partial and more noisy. Data for later years had not yet been released at the time of writing. The original dataset contains 55889 observations for 20144 different firms. I do several adjustments on the raw data, listed below.

1. I delete observations which do not belong to industries targeted by the MIP, using the definition of the MIP's scope provided in Rammer et al. (2011). I also drop observations with missing or incorrect NACE codes and, following a usual convention in firm-level studies, the financial industry (NACE codes 650 to 672) and the energy and water industry (NACE codes 400 to 410).

2. I delete observations with missing or zero values for employment (variable bges) or sales (variable um).[65]

3. I delete several outliers. In some firms, the unit of reference may change over time (a subsidiary may report sometimes its own values, and sometimes values for the mother company). This creates large jumps in firm-level time series. To avoid them, I drop for every firm observations with sales or employment which are two times larger or two times smaller than the firm's median for the respective variable. This excludes around 5% of all observations. Note that this does not eliminate outliers for firms with only one observation. However, these are irrelevant for my analysis, as they are dropped by definition in any first difference analysis or panel estimation. I additionally control for outliers by winsorizing the variables used in regressions, as indicated in the main text.

After these adjustments, the dataset contains 41874 observations for 16506 different firms. Table 8 shows that my dataset contains between 2700 and 5800 firms per year. There are systematically more observations

[64] This includes all manufacturing industries, mining and some services (wholesale trade, transport, software development and services to firms). It excludes agriculture, construction and services such as retail trade, real estate or health care.

[65] I also fill in some missing values. If a firm reports in a survey that it never did R&D over the last three years (variable fuekon equal to 0) and has a missing value for R&D in that year, I replace the missing value by a 0. I do the same for non-innovating firms which have a missing value for innovation spending. Non-innovating firms are firms which have had no product or process innovation, no current innovation project and no interrupted innovation project in the last three years (variables pd, pz, pa and pn all equal to 0) or firms which have not spent on innovation projects (variable iagesno equal to 1).

in even years, where the ZEW refreshes the sample. There have been some changes in the scope of the survey over time, so that cross-sectional statistics in different years are in general not directly comparable. However, Table 8 indicates that the share of SMEs and of innovating firms in the sample is relatively stable over time. The share of SMEs increases somewhat with the enlargement of the sample, and therefore, the share of innovating firms falls (as SMEs are less likely to do R&D than large firms).

The MIP survey asks firms about their R&D spending ("Forschungs- und experimentelle Entwicklungsausgaben") and innovation spending ("Innovationsausgaben"). R&D is defined as "*the systematic creative work for the enlargement of current knowledge and the use of the so gained knowledge to develop new products or processes*" (Rammer et al. (2011), my translation). While this variable is widely used in the empirical literature, innovation spending is a less common concept. It covers, on top of R&D, several types of implementation spending.[66] In the stylized framework of my model, innovation spending and R&D investment coincide. In reality, innovation spending may capture some innovative activities ignored by the standard R&D measure. As indicated in the main text and shown in Section B.2, the behaviour of R&D and innovation spending in small and large firms over the Great Recession is in general quite similar.

Table 8: Summary statistics

Year	Observations	Share of SMEs	Share of inn. firms
1999	2704	84%	49%
2000	3370	90%	49%
2001	2734	85%	n.a.
2002	3308	87%	56%
2003	3091	88%	57%
2004	3879	90%	53%
2005	3506	89%	55%
2006	3908	91%	52%
2007	4808	91%	51%
2008	5785	92%	44%
2009	4781	92%	48%
Total	**41874**	**90%**	**50%**

Notes: Innovating firms are firms with at least one non-zero observation for R&D. In 2001, the survey had no question on R&D.

[66]This includes buying machines to produce a new product, marketing expenses, patent or licence purchases, spending on continuing education of employees, etc.

B.2 Additional Tables

This section contains the additional tables referred to in the main text in Section 4.1. Tables 9 and 10 show the mean rates of change of R&D and sales by firm size during the Great Recession, unconditional and conditional on the sign of sales changes, and Table 11 contains the (unconditional) median rates of change for innovation spending. These tables globally confirm the picture suggested in the main text, except that the mean changes of R&D in 2008 do not show a greater reaction of small firms, due to some outliers among large firms. In 2009, however, differences remain very salient.

Table 9: R&D and sales fluctuations by firm size during the Great Recession, Means

	2008			**2009**		
Mean rate of change	**R&D**	**Sales**	**Obs.**	**R&D**	**Sales**	**Obs.**
Small (50 or less emp.)	3.37	6.83	283	-10.95	-4.48	363
Large (more than 500 emp.)	-3.56	1.19	121	-4.19	-15.77	103

Notes: See notes to Table 2.

Table 10: R&D fluctuations conditional on the sign of sales changes, Means

	2008			**2009**		
Mean rate of change	**R&D**	**Sales**	**Obs.**	**R&D**	**Sales**	**Obs.**
Firms with decreasing sales						
Small (50 or less emp.)	-24.11	-15.15	87	-27.15	-22.01	201
Large (more than 500 emp.)	-26.79	-14.47	45	-9.88	-26.01	76
Firms with increasing sales						
Small (50 or less emp.)	15.58	16.59	196	9.15	17.28	162
Large (more than 500 emp.)	10.20	10.46	76	11.81	13.03	27

Notes: See notes to Table 2. Firms with unchanged sales are included in the increasing sales category.

Table 11: Innovation spending and sales fluctuations by firm size during the Great Recession

	2008			2009		
Median rate of change	**Inn. Sp.**	**Sales**	**Obs.**	**Inn. Sp.**	**Sales**	**Obs.**
Small (50 or less emp.)	-5.79	5.41	516	-8.86	-4.40	647
Large (more than 500 emp.)	0.00	2.14	148	0.00	-13.68	137

Notes: See notes to Table 2. Note that innovating firms are now defined with respect to the innovation spending variable (that is, the sample includes firms if and only if they have at least one positive observation for innovation spending) and that firms with trivial innovation spending (less than 10.000€) are excluded.

Table 12 reproduces Table 4 using industry-year fixed effects (at the NACE two-digit level).

Table 12: Panel regressions with industry-year fixed effects

Dependent variable: R&D intensity		
Sample	All firms	Res.-int. manuf.
SE	$-1.11\,(0.53)^{**}$	$-2.17\,(0.93)^{**}$
SED^{1999}	$1.41\,(0.58)^{**}$	$2.06\,(1.07)^{*}$
SED^{2000}	$1.11\,(0.51)^{**}$	$3.15\,(0.87)^{***}$
SED^{2002}	$2.27\,(0.50)^{***}$	$4.40\,(0.89)^{***}$
SED^{2003}	$1.32\,(0.47)^{***}$	$3.02\,(0.88)^{***}$
SED^{2004}	$1.79\,(0.45)^{***}$	$3.04\,(0.96)^{***}$
SED^{2005}	$0.73\,(0.43)^{*}$	$1.68\,(0.80)^{**}$
SED^{2006}	$0.78\,(0.40)^{*}$	$1.87\,(0.82)^{**}$
SED^{2007}	$0.81\,(0.41)^{**}$	$2.44\,(0.83)^{**}$
SED^{2008}	$0.70\,(0.41)^{**}$	$1.37\,(0.70)^{*}$
Obs.	15206	5729

Notes: See Table 4. Note that the R&D services industry is now included, as industry-level differences in the elasticity of R&D with respect to sales are fully accounted for by industry-year fixed effects.

Finally, Table 13 is the equivalent of Table 4, using innovation intensity as the dependent variable.

Table 13: Panel regressions with innovation intensity

Dependent variable: Innovation intensity

Sample	All firms	Res.-int. manuf.
SE	$-1.95\,(0.62)^{***}$	$-2.36\,(1.05)^{**}$
SED^{1999}	$2.26\,(0.65)^{***}$	$1.97\,(1.12)^{*}$
SED^{2000}	$1.44\,(0.55)^{***}$	$3.25\,(0.95)^{***}$
SED^{2001}	$1.78\,(0.60)^{***}$	$1.73\,(1.06)$
SED^{2002}	$1.70\,(0.60)^{***}$	$2.64\,(0.99)^{***}$
SED^{2003}	$2.36\,(0.57)^{***}$	$3.47\,(0.93)^{***}$
SED^{2004}	$2.63\,(0.51)^{***}$	$2.87\,(0.92)^{***}$
SED^{2005}	$1.80\,(0.47)^{***}$	$1.55\,(0.82)^{*}$
SED^{2006}	$0.99\,(0.49)^{**}$	$0.91\,(0.86)$
SED^{2007}	$1.63\,(0.45)^{**}$	$1.70\,(0.81)^{**}$
SED^{2008}	$0.95\,(0.45)^{**}$	$-0.08\,(0.77)$
Obs.	23554	7022

Notes: See Table 4. There are now 6351 firm-level and 384 industry-year level clusters in the full sample (2011 and 88 in the research-intensive manufacturing sample). Note that innovating firms are now defined with respect to the innovation spending variable.

B.3 Industry-level Productivity data

In Section 4.3.1, I measure labour productivity at the industry level as real shipments per employee. Real shipments are defined as $\frac{\text{vship}}{\text{piship}}$ (using the variable names of the NBER-CES database) and employees are given by emp. I limit my analysis to the period 1972-2009, as data on capacity utilisation, used as a control variable in regressions, is only available from 1972 onwards.

As regressions are specified in natural logarithms, I need to omit industries which had negative average labour productivity growth from 1972 to 2009. This is the case only for 5 small industries: Manufactured Home (Mobile Home) Manufacturing (NAICS Code 321991), Prefabricated Wood Building Manufacturing (321992), All Other Leather Good and Allied Product Manufacturing (316999), Coffee and Tea Manufacturing (311920) and Prefabricated Metal Building and Component Manufacturing (332311).